Intrinsic Spin and Orbital Angular Momentum Hall Effect

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(Received 27 July 2004; published 17 February 2005)

A generalized definition of intrinsic and extrinsic transport coefficients is introduced. We show that transport coefficients from the intrinsic origin are solely determined by local electronic structure, and thus the intrinsic spin Hall effect is not a transport phenomenon. The intrinsic spin Hall current is always accompanied by an equal but opposite intrinsic orbital angular momentum Hall current. We prove that the intrinsic spin Hall effect does not induce a spin accumulation at the edge of the sample or near the interface.

DOI: 10.1103/PhysRevLett.94.066602 PACS numbers: 72.10.–d, 72.15.Gd, 73.50.Jt

Recently, there are emerging *theoretical* interests on the spin Hall effect in a spin-orbit coupled system [1–14]. The spin Hall effect refers to a nonzero *spin* current in the direction transverse to the direction of the applied electric field. Earlier studies had been focused on an extrinsic effect [15,16], namely, when conduction electrons scatter off an impurity with the spin-orbit interaction, the electrons tend to deflect to the left (right) more than to the right (left) for a given spin orientation of the electrons. Thus the impurity is the prerequisite in the extrinsic spin Hall effect. Recently, the spin Hall effect has been extended to semiconductor heterostructures where the spin-orbit coupled *bands* are important. It has been shown that the spin current exists in the absence of impurities, termed as the intrinsic or dissipationless spin Hall effect (ISHE) in order to distinguish the impurity-driven extrinsic spin Hall effect (ESHE) mentioned above. In general, the magnitude of ISHE is two to three orders larger than that of ESHE; this immediately generates an explosive interest in theoretical research on the ISHE since the spin current is regarded as one of the key variables in spintronics application.

However, the spin current generated via ISHE is fundamentally different from conventional spin-polarized transport in many ways. First, the spin current is carried by the entire spin-orbit coupled Fermi sea, not just electrons or holes at the Fermi level [1,4]. Second, ISHE exists even for an equilibrium system (without external electric fields) [5] and ISHE is closely related to the dielectric response function that characterizes the electronic deformation [6]. Most recently, it is proposed that the intrinsic spin Hall effect exists even in insulators [17]. The above unconventional properties cast serious doubts on experimental relevance of the intrinsic spin current. It has been already alerted by Rashba [5,6] that the ISHE may not be a transport phenomenon. Because of the ill-defined nature of the spin current in the spin-orbit coupled Hamiltonian, theories utilizing different approaches produce contradicting results: some predicted a zero spin Hall current in the presence of an arbitrary weak disorder and some claimed a universal spin conductivity at weak disorder. In this Letter, we do not try to resolve the above theoretical debate, instead we reveal the spurious nature of the intrinsic spin Hall effect and discuss its experimental consequences. We first define generalized intrinsic and extrinsic transport coefficients from the semiclassical transport equation. We show that the intrinsic spin current is always accompanied by an equal but opposite orbital angular momentum (OAM) current for a spin-orbit coupled system. Thus, the intrinsic magnetization current which is the sum of the spin current and the orbital angular momentum current is identically zero. Next, we construct the equation of motion for the spin density in the presence of the intrinsic and extrinsic mechanisms. We find that the intrinsic spin current is exactly canceled by a spin torque and thus the spin accumulation at the edge of the sample is solely determined by the extrinsic spin current. The above results make us conclude that the intrinsic spin current has *no experimental consequences* in terms of the spin transport measurement for an arbitrary strength of the intrinsic spin Hall conductivity. Therefore, the intrinsic spin current is a pure theoretical object, at least, in the limit of the semiclassical picture of the spin transport. Finally, we brief comment on the most recent experimental results [18].

Let us consider a spin-dependent Hamiltonian

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}, \sigma) + eEx + V_i(\mathbf{r}, \sigma) \qquad (1)$$

where the second term represents a periodic spin-orbit potential, the third term is the interaction with a dc electric field E in the x direction, and the last term is the impurity potential that may or may not depend on spin. Now let us consider how an arbitrary dynamic variable $\hat{G}$ responds to the electric field. A standard semiclassical version of the linear response function $\mathcal{J}(\mathbf{r})$ is

$$\mathcal{J}(\mathbf{r}) = \sum_{\mathbf{k}\lambda} G_{\mathbf{k}\lambda} f(\epsilon_{\mathbf{k}\lambda}, \mathbf{r}) \qquad (2)$$

where $\hat{G}$ can be any dynamic variable such as the current density, the spin current density, the magnetic moment, etc.; $G_{\mathbf{k}\lambda} = \int d\mathbf{r}\Psi^+_{\mathbf{k}\lambda}(\mathbf{r})\hat{\mathbf{G}}\Psi_{\mathbf{k}\lambda}(\mathbf{r}) \equiv \langle\Psi_{\mathbf{k}\lambda}|\hat{\mathbf{G}}|\Psi_{\mathbf{k}\lambda}\rangle$ is the expectation value for the eigenstate $\Psi_{\mathbf{k}\lambda}(\mathbf{r})$ (Bloch states) determined by the first three terms in Eq. (1); $\lambda = \pm 1$ represents the index of the spin sub band; and $f(\epsilon_{\mathbf{k}\lambda}, \mathbf{r})$ is the distribution function that depends on the detail of the scattering potential $V_i(\mathbf{r}, \sigma)$, the last term of Eq. (1). The dependence of $J(\mathbf{r})$ on the electric field enters in two places: the wave functions and the distribution function. We may expand them up to the first order in the electric field. The wave function is written as,

$$\Psi_{\mathbf{k}\lambda}(\mathbf{r}) = \Psi^{(0)}_{\mathbf{k}\lambda}(\mathbf{r}) + \Psi^{(1)}_{\mathbf{k}\lambda}(\mathbf{r}) \tag{3}$$

where $\Psi^{(0)}_{\mathbf{k}\lambda}(\mathbf{r})$ is the unperturbed electronic structure determined by the first two terms in Eq. (1), and

$$\Psi^{(1)}_{\mathbf{k}\lambda}(\mathbf{r}) = \sum_{\mathbf{k}'\lambda' \neq \mathbf{k}\lambda} \frac{\langle \Psi^{(0)}_{\mathbf{k}'\lambda'} | eEx | \Psi^{(0)}_{\mathbf{k}\lambda} \rangle}{\epsilon_{\mathbf{k}\lambda} - \epsilon_{\mathbf{k}'\lambda'}} \Psi^{(0)}_{\mathbf{k}'\lambda'}(\mathbf{r}), \tag{4}$$

is the first order perturbation to the third term in Eq. (1). Similarly, we write the distribution function in terms of the equilibrium and nonequilibrium parts,

$$f(\epsilon_{\mathbf{k}\lambda}, \mathbf{r}) = f^0(\epsilon_{\mathbf{k}\lambda}) + \left(-\frac{\partial f^0}{\partial \epsilon_{\mathbf{k}\lambda}}\right) g(\mathbf{k}\lambda, \mathbf{r}) \tag{5}$$

where f^0 is the equilibrium distribution function and the nonequilibrium function $g(\mathbf{k}\lambda, \mathbf{r})$ is proportional to the electric field. By placing Eqs. (3) and (5) into Eq. (2) and keeping only the first order term in the electric field, we have $J(\mathbf{r}) \equiv J_{\rm int} + J_{\rm ext}$ where

$$J_{\rm int} = 2{\rm Re} \sum_{\mathbf{k}\lambda} \langle \Psi^{(0)}_{\mathbf{k}\lambda} | \hat{G} | \Psi^{(1)}_{\mathbf{k}\lambda} \rangle f^0(\epsilon_{\mathbf{k}\lambda}) \tag{6}$$

is defined as the intrinsic linear response and Re stands for the real part, and

$$J_{\rm ext} = \sum_{\mathbf{k}\lambda} \langle \Psi^{(0)}_{\mathbf{k}\lambda} | \hat{G} | \Psi^{(0)}_{\mathbf{k}\lambda} \rangle \left(-\frac{\partial f^0}{\partial \epsilon_{\mathbf{k}\lambda}}\right) g(\mathbf{k}\lambda, \mathbf{r}) \tag{7}$$

is called the extrinsic linear response. The above distinction between intrinsic and extrinsic contributions to the transport properties has been already introduced by a number of groups, in particular, by Jungwirth *et al.* [19] in their study of the anomalous Hall effect in itinerant ferromagnets. Equation (6) shows that the intrinsic linear response coefficient is not related to the transport phenomenon since $J_{\rm int}$ is determined by the equilibrium distribution function and the *local electronic structure*. Thus, there are no transport length scales such as the mean free path or spin-diffusion length in $J_{\rm int}$. The extrinsic linear response, $J_{\rm ext}$, is a true transport quantity because it is directly proportional to the nonequilibrium distribution function that is determined by various scattering mechanisms. At low temperature, the factor of $\partial f^0/\partial \epsilon_{\mathbf{k}\lambda}$ limits the transport states to the Fermi level. Comparing Eqs. (6) and (7), we realize that the intrinsic effect is simple and easy to calculate while the extrinsic effect is much more complicated. As long as we know the Bloch states $\Psi^{(0)}_{\mathbf{k}\lambda}$, the intrinsic transport coefficient can be straightforwardly evaluated since f^0 is known. The extrinsic transport coefficient not only depends on the Bloch states, but also on the nonequilibrium distribution function that is usually the center of the relevant physics. Here, however, we should concentrate on the easy problem: calculation of the intrinsic transport from Eq. (6) by using a model Hamiltonian.

We choose a Rashba Hamiltonian to illustrate the physics of the intrinsic transport properties. A similar calculation can also be performed for a Luttinger Hamiltonian [20]. For the Rashba Hamiltonian, the second term of Eq. (1) is $V(\mathbf{r}, \sigma) = (\alpha/\hbar)\sigma \cdot (\mathbf{p} \times \hat{\mathbf{z}})$ where $\mathbf{p}$ is the momentum in the xy plane, σ is the Pauli matrix, and α is the coupling constant. Before we calculate the spin and the OAM Hall currents from Eq. (6), we list the wave function and the dispersion relation of the Rashba Hamiltonian so that one can easily follow our derivation at each step

$$\Psi^{(0)}_{\mathbf{k}\lambda}(\mathbf{r}) = \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{2A}} \begin{pmatrix} 1 \\ -i\lambda(k_x + ik_y)k^{-1} \end{pmatrix} \tag{8}$$

where A is the area of the 2-dimensional electron gas, and

$$\epsilon_{\mathbf{k}\lambda} = \frac{\hbar^2 k^2}{2m} + \lambda \alpha k \tag{9}$$

where $k = |\mathbf{k}| = \sqrt{k_x^2 + k_y^2}$. By placing the above two equations into (4), we have [5]

$$\Psi^{(1)}_{\mathbf{k}\lambda}(\mathbf{r}) = -\frac{\lambda e E k_y}{4\alpha k^3} \Psi^{(0)}_{\mathbf{k}-\lambda}(\mathbf{r}). \tag{10}$$

In obtaining the above result, we have used $\langle \Psi^{(0)}_{\mathbf{k}'\lambda'} | x | \Psi^{(0)}_{\mathbf{k}\lambda} \rangle = -\frac{\lambda\lambda'}{2} \delta_{\mathbf{k}\mathbf{k}'} \frac{k_y}{k^2}$.

We now proceed to calculate the spin Hall current by taking the operator $\hat{G} = (1/2)[s_z v_y + v_y s_z]$ where $s_z = (\hbar/2)\sigma_z$ is the z component of the spin operator and v_y is the y component of the velocity. By placing the above definition along with Eqs. (8) and (10), and by taking the distribution function a step function at zero temperature, we obtain the intrinsic spin Hall current from Eq. (6),

$$J^{\rm spin}_{\rm int} = \frac{e}{8\pi} E; \tag{11}$$

where we have assumed that the Fermi energy is larger than the spin-orbit coupling energy so that both spin subbands cross the Fermi level. Equation (11) represents the universal spin conductivity $(e/8\pi)$ obtained by many groups [1,4].

Our central question is: what is the physical meaning of this spin current derived from the equilibrium distribution function? To see clearly what this spin current represents, we recall that the spin is not a conserved quantity in a spin-orbit coupled system. If one is interested in the *magnetization current* or the total angular momentum current, one should also include the OAM Hall current. The OAM Hall current can be similarly calculated by introducing an operator for the OAM Hall current

$$\hat{G} = \frac{1}{2}[(\mathbf{r}\times\mathbf{p})_z v_y + v_y(\mathbf{r}\times\mathbf{p})_z] \tag{12}$$

where $(\mathbf{r}\times\mathbf{p})_z = xp_y - yp_x$ is the z component of the OAM. The same straightforward evaluation of Eq. (6) leads to

$$J_{\mathrm{int}}^{\mathrm{orbit}} = -\frac{e}{8\pi}E. \tag{13}$$

Thus the OAM current is exactly equal and opposite to the spin current. This result is not surprising at all: the total angular momentum (z component), spin plus orbital is conserved for the Rashba Hamiltonian and thus we can choose the Bloch states that are simultaneous eigenstates of the total angular momentum and the Hamiltonian. In fact, one can directly show that the total angular momentum current vanishes if we use $[s_z + L_z, H] = 0$ to the Rashba Hamiltonian.

Having discussed that the intrinsic spin current is always accompanied with the OAM current in a bulk spin-orbit coupled material, our next question is whether the intrinsic spin current can produce a spin accumulation at the edge of the sample? To answer this question, we recall the basic idea of the spin accumulation for the extrinsic spin current. When an extrinsic spin current spatially varies, nonequilibrium spins will be accumulated so that the spin diffusion is balanced by the spin-drift current. Equivalently, the spin accumulation results in the chemical potential splitting between two spin subbands and a voltage can be measured experimentally when the sample is attached to a ferromagnetic lead [16]. Mathematically, the nonequilibrium spin-dependent chemical potential or spin accumulation is the average of the *nonequilibrium distribution function* [21]. For the intrinsic spin Hall current, the distribution is an equilibrium distribution and one would expect that the concept of the spin-dependent chemical potential breaks down. Indeed, we show next that the intrinsic spin Hall current does not lead to spin accumulations at the sample edge and across an interface.

To calculate the spin accumulation or the position-dependent spin density $\mathbf{S}(\mathbf{r}, t)$ at the edge of the sample or across an interface, one relies on the semiclassical equation of motion that can be generally written as

$$\frac{\partial \mathbf{S}(\mathbf{r}, t)}{\partial t} + \nabla\cdot[\mathbf{J}_{\mathrm{int}} + \mathbf{J}_{\mathrm{ext}}] = \tau_{\mathrm{int}} + \tau_{\mathrm{ext}} + \left(\frac{\partial \mathbf{S}}{\partial t}\right)_{\mathrm{colli}} \tag{14}$$

where $\mathbf{J}_{\mathrm{int}}$ and $\mathbf{J}_{\mathrm{ext}}$ are the intrinsic and extrinsic spin current densities, τ_{int} and τ_{ext} are the intrinsic and extrinsic spin torques due to noncommutivity of the Hamiltonian with the spin operator, i.e., the spin torques are calculated by replacing $\hat{G}$ by $[\mathbf{s}, H]/i\hbar$ in Eqs. (6) and (7), and the last term in Eq. (14) is a collision term that is to relax the nonequilibrium distribution function to an equilibrium one. To explicitly obtain the spin accumulation $\mathbf{S}(\mathbf{r}, t)$ in a closed form, it is necessary to use a wave-package description so that the position dependence can be readily included. Culcer *et al.* [2] have already formulated that the intrinsic spin torque τ_{int} can be written as two terms. In our notation, we find $\tau_{\mathrm{int}} = \tau_0 + \tau_1$ where

$$\tau_0 = \sum_{\mathbf{k}\lambda}\langle\mathbf{k}\lambda|\frac{1}{i\hbar}[\mathbf{s}, H]|\mathbf{k}\lambda\rangle f^0(\epsilon_{\mathbf{k}\lambda}) \tag{15}$$

and

$$\tau_1 = -\nabla\cdot\sum_{\mathbf{k}\lambda}\langle\mathbf{k}\lambda|\frac{1}{i\hbar}[\mathbf{s}, H]\mathbf{r}|\mathbf{k}\lambda\rangle f^0(\epsilon_{\mathbf{k}\lambda}) \tag{16}$$

where the symmetrization of the product of $[\mathbf{s}, H]$ and $\mathbf{r}$ is implied. We emphasize that τ_1 comes from the position dependence of the center of the wave packet. By using the fact that the wave function is an eigenstate of the Hamiltonian $H|\mathbf{k}\lambda\rangle = E_{\mathbf{k}\lambda}|\mathbf{k}\lambda\rangle$ we immediately see that the expectation value of $[\mathbf{s}, H]$ is zero, i.e., $\tau_0 = 0$. To calculate τ_1, we use the commuting relation $[H, \mathbf{r}] = -i\hbar\mathbf{v} + i\alpha(\mathbf{e}_z\times\sigma)$, where $\mathbf{v}$ is the velocity operator. After a straightforward algebra simplification, we have found [22]

$$\tau_{\mathrm{int}} = \nabla\cdot\sum_{\mathbf{k}\lambda}\langle\mathbf{k}\lambda|\frac{\mathbf{vs}+\mathbf{sv}}{2}|\mathbf{k}\lambda\rangle f^0(\epsilon_{\mathbf{k}\lambda}) \equiv \nabla\cdot\mathbf{J}_{\mathrm{int}}. \tag{17}$$

Therefore, the spin torque exactly equals the divergence of the spin current. The equation of motion, Eq. (14), now becomes

$$\frac{\partial \mathbf{S}(\mathbf{r}, t)}{\partial t} + \nabla\cdot\mathbf{J}_{\mathrm{ext}} = \tau_{\mathrm{ext}} + \left(\frac{\partial \mathbf{S}}{\partial t}\right)_{\mathrm{colli}}. \tag{18}$$

We conclude that the intrinsic spin current does not enter into the play in the equation of motion. The spin accumulation $\mathbf{S}(\mathbf{r}, t)$ is solely determined by the extrinsic part of the current density.

We now return to our central issue on the problem, namely, whether the intrinsic spin Hall conductivity can be measured via conventional meanings of the spin transport. Since the spin current is not directly measurable, two schemes are usually employed: one is the realization of measuring the electric field induced by the *magnetization current* [23] and the other is the spin accumulation at the sample of the edge or across an interface [24]. We should discuss them separately below.

If there is a net magnetization current in a bulk material, a circular electric field outside the sample will be induced. This phenomenon is analogous to the magnetic field induced by a charge current, known as the Biot-Savart law or Ampère's law. For example, it was proposed that a spin current generated via spin wave propagation through a nanowire can be detected by an induced electric field just outside the nanowire [23,25]. However, the above proposal is applied to the case where the OAM current is absent. In the present case, the OAM current is exactly opposite to the spin current so that the net magnetization current is zero. Therefore, we conclude that there is no electric field associated with the intrinsic spin current.

The more efficient method to detect the spin current is by measuring the spin accumulation due to spatial variation of the spin current, e.g., the Johnson-Silsbee's experiment [24]. Based on the equation of motion given by Eq. (18), the divergent of the *extrinsic but not intrinsic* spin currents can lead to a buildup of spin accumulation. To determine the spin accumulation, one usually makes a relaxation-time approximation so that the collision term in Eq. (18) is modeled by $-\mathbf{S}/\tau_{sf}$. Since the intrinsic spin current does not contribute to the equation of motion for the spin accumulation, the measurement based on the detection of the spin accumulation will produce a null contribution from the spin Hall effect, no matter how large the intrinsic spin current is.

Two experimental groups have recently observed the spin Hall effect by detecting spin accumulation at the edges of the samples [18]. Kato *et al.* argued that the effect is extrinsic based on their experimental results that the spin accumulation is independent of the strain direction, while Wunderlich *et al.* claimed that their observed effect is intrinsic based on the assumption that the impurity scattering is weaker than the spin-orbit coupling in their samples (clean limit). We point our here that the clean limit in the experiment does not imply the spin accumulation from the intrinsic origin. Instead, our analysis has shown that no matter how large is the intrinsic spin current, the observed effect has to be an extrinsic origin because the spin accumulation is independent of the intrinsic spin current.

We finally draw a picture on why the intrinsic spin Hall fails to produce experimental consequences. Consider a contact between a Rashba material and a nonmagnetic material with no spin-orbit coupling. The spin current, as well as the orbital angular momentum current, would exist in the Rashba material. However, the spin current drops to zero across the interface of the nonmagnetic material, i.e., the spin current is not continuous; this is because the spin torque produces a mechanism to transfer the spin current to the orbital angular momentum current or vice versus. As a result, when the spin-orbit coupling vanishes at the non-spin-orbit coupled material, both the spin and orbital angular momentum currents drop to zero. The loss of the spin current exactly equals to the gain of the OAM current so that the total angular momentum current or magnetization current is continuous across the interface of the layers; they are both zero. For the same reason, the edge of the sample never develops spin accumulation because the usual boundary condition of zero spin current at the surface is no more valid, instead, the total angular momentum current is zero at the surface for the intrinsic spin Hall effect.

In conclusion, we have constructed a general framework for calculating intrinsic linear response coefficients. We have shown that the intrinsic spin Hall effect is accompanied by the intrinsic orbital angular momentum Hall effect so that the magnetization current is zero in a spin-orbit coupled system. The intrinsic spin Hall effect is not a useful source of spin currents because the intrinsic spin current does not enter into the equation of motion for the spin transport. Most of the proposed experimental detections of the intrinsic spin Hall effect are the artifact of the boundary conditions that are not valid for the intrinsic spin Hall current.

This work is partially supported by DARPA-SPINS and by NSF-DMR-0314456.

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