Theory of Current-Driven Domain Wall Motion: Spin Transfer versus Momentum Transfer

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A self-contained theory of the domain wall dynamics in ferromagnets under finite electric current is presented. The current has two effects: one is momentum transfer, which is proportional to the charge current and wall resistivity (ρ_w); the other is spin transfer, proportional to spin current. For thick walls, as in metallic wires, the latter dominates and the threshold current for wall motion is determined by the hard-axis magnetic anisotropy, except for the case of very strong pinning. For thin walls, as in nanocontacts and magnetic semiconductors, the momentum-transfer effect dominates, and the threshold current is proportional to V_0/ρ_w, V_0 being the pinning potential.

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Manipulation of magnetization and magnetic domain wall [1] by use of electric current is of special interest recently [2–6], from the viewpoint of application to spintronics, e.g., novel magnetic devices where the information is written electrically, and also as a basic physics in that it involves fascinating angular momentum dynamics.

Current-driven motion of a domain wall was studied in a series of pioneering works by Berger [7–9]. In 1984, he argued that the electric current exerts a force on the domain wall via the exchange coupling [8]. Later, in 1992, he discussed that a spin-polarized current (spin current) exerts a torque on the wall magnetization and studied the wall motion due to a pulsed spin-polarized current [9]. These theoretical works are based on his deep physical insight but seem to lack transparency as a self-contained theory. Also, their phenomenological character makes the limit of applicability unclear. In view of recent precise experiments [4–6], a general theory starting from a microscopic description is now needed.

In this Letter, we reformulate the problem of domain wall dynamics in the presence of electric current and explore some new features such as current-induced depinning of the wall. We start from a microscopic Hamiltonian with an exchange interaction between conduction electrons and spins of a domain wall [10]. With a key observation that the wall position X and polarization ϕ_0 (the angle between spins at the wall center and the easy plane) are the proper collective coordinates [11] to describe its dynamics, it follows straightforwardly that the electric current affects the wall motion in two different ways, in agreement with Berger's observation. The first is as a force on X, or momentum transfer, due to the reflection of conduction electrons. This effect is proportional to the charge current and wall resistance and, hence, is negligible except for very thin walls. The other is as a spin torque (a force on ϕ_0), arising when an electron passes through the wall. Nowadays it is also called as spin transfer [2] between electrons and wall magnetization. This effect is the dominant one for thick walls where the spin of the electron follows the magnetization adiabatically.

The motion of a domain wall under a steady current is studied in two limiting cases. In the adiabatic case, we show that even without a pinning force, there is a threshold spin current j_s^{cr} below which the wall does not move. This threshold is proportional to $K_\perp$, the hard-axis magnetic anisotropy. Underlying this is that the angular momentum transferred from the electron can be carried by both X and ϕ_0, and the latter can completely absorb the spin transfer if the spin current is small, $j_s < j_s^{cr}$. The pinning potential V_0 affects j_s^{cr} only if it is very strong, $V_0 \gtrsim K_\perp/\alpha$, where α is the damping parameter in the Landau-Lifshits-Gilbert equation. In most real systems with small α, the threshold would thus be determined by $K_\perp$. Therefore, the critical current for the adiabatic wall will be controllable by the sample shape and, in particular, by the thickness of the film and does not suffer very much from pinning arising from sample irregularities. This would be a great advantage in application. The wall velocity after depinning is found to be $\langle \dot{X} \rangle \propto \sqrt{(j_s/j_s^{cr})^2 - 1}$.

In the case of a thin wall, the wall is driven by the momentum transfer, which is proportional to the charge current j and wall resistivity ρ_w. The critical current density in this case is given by $j^{cr} \propto V_0/\rho_w$.

We consider a ferromagnet consisting of localized spins $\boldsymbol{S}$ and conduction electrons. The spins are assumed to have an easy z axis and a hard y axis. In the continuum approximation, the spin part is described by the Lagrangian [12–14]

$$L_S = \int \frac{d^3x}{a^3}\Big[\hbar S\dot{\phi}(\cos\theta - 1) - V_{\rm pin}[\theta] - \frac{S^2}{2}\{J((\nabla\theta)^2 + \sin^2\theta(\nabla\phi)^2) + \sin^2\theta(K + K_\perp \sin^2\phi)\}\Big], \quad (1)$$

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where a is the lattice constant, and we put $\boldsymbol{S}(x) = S(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, and J represents the exchange coupling between localized spins. The longitudinal (K) and transverse ($K_\perp$) anisotropy constants incorporate the effect of demagnetizing field. The constants J, K, and $K_\perp$ are all positive. The term $V_{\rm pin}$ represents pinning due to additional localized anisotropy energy. The exchange interaction between localized spins and conduction electrons is given by

$$H_{\rm int} = -\frac{\Delta}{S}\int d^3x \boldsymbol{S}(x)(c^\dagger \boldsymbol{\sigma} c)_x, \qquad (2)$$

where 2Δ and c ($c^\dagger$) are the energy splitting and annihilation (creation) operator of conduction electrons, respectively, and $\boldsymbol{\sigma}$ is a Pauli-matrix vector. The electron part is given by $H_{\rm el} = \sum_{\boldsymbol{k}} \epsilon_{\boldsymbol{k}} c_{\boldsymbol{k}}^\dagger c_{\boldsymbol{k}}$ with $\epsilon_{\boldsymbol{k}} = \hbar^2\boldsymbol{k}^2/2m$.

In the absence of $V_{\rm pin}$ and $H_{\rm int}$, the spin part has a static domain wall of width $\lambda \equiv (J/K)^{1/2}$ as a classical solution. We consider a wire with width smaller than λ and treat the spin configuration as uniform in the yz plane, perpendicular to the wire direction x. The solution centered at $x = X$ is given by $\theta = \theta_0(x - X)$, $\phi = 0$, where $\cos\theta_0(x) = \tanh(x/\lambda)$, and $\sin\theta_0(x) = (\cosh(x/\lambda))^{-1}$. To describe the dynamics of the domain wall, it is crucial to observe that the weighted average of ϕ, defined by $\phi_0(t) \equiv \int(dx/2\lambda)\phi(x,t)\sin^2\theta_0(x - X(t))$ plays the role of momentum conjugate to X and, hence, must be treated as dynamical [14]. Neglecting spin-wave excitations, we obtain the Lagrangian for $X(t)$ and $\phi_0(t)$ as

$$L_S = -\frac{\hbar NS}{\lambda}X\dot{\phi}_0 - \frac{1}{2}K_\perp NS^2\sin^2\phi_0 - V_{\rm pin}(X), \qquad (3)$$

where $V_{\rm pin}(X)$ is a pinning potential for X, and $N = 2A\lambda/a^3$ is the number of spins in the wall. (A is the cross-sectional area.) The equations of motion, derived from the Lagrangian, $L_S - H_{\rm int}$, are given by

$$\frac{\hbar NS}{\lambda}\left(\dot{\phi}_0 + \alpha\frac{\dot{X}}{\lambda}\right) = F_{\rm pin} + F_{\rm el}, \qquad (4)$$

$$\frac{\hbar NS}{\lambda}(\dot{X} - \alpha\lambda\dot{\phi}_0) = \frac{NS^2K_\perp}{2}\sin 2\phi_0 + T_{{\rm el},z}, \qquad (5)$$

where $F_{\rm pin} = -(\partial V_{\rm pin}/\partial X)$,

$$F_{\rm el} \equiv -\frac{\Delta}{S}\int d^3x \nabla_x \boldsymbol{S}_0(x - X)\cdot \boldsymbol{n}(x), \qquad (6)$$

$$\boldsymbol{T}_{\rm el} \equiv -\frac{\Delta}{S}\int d^3x \boldsymbol{S}_0(x - X)\times \boldsymbol{n}(x). \qquad (7)$$

Here $\boldsymbol{S}_0$ denotes $\boldsymbol{S}(x)$ with $\theta = \theta_0(x - X)$, $\phi = \phi_0$, and $n_\mu \equiv \langle c^\dagger\sigma_\mu c\rangle$ ($\mu = x, y, z$) is (twice) the spin density of conduction electrons. $F_{\rm el}$ represents a force acting on the wall, or momentum transfer, due to the electron flow, while $\boldsymbol{T}_{\rm el}$ is a spin torque, or spin transfer, which comes from the directional mismatch between wall magnetization $\boldsymbol{S}_0(x - X)$ and $\boldsymbol{n}(x)$. We have added a damping term (α), which represents a standard damping torque (Gilbert damping), $\boldsymbol{T}_{\rm damp} = -\frac{\alpha}{S}\boldsymbol{S}\times\dot{\boldsymbol{S}}$ [1]. Note that the spin-transfer effect acts as a source to the wall velocity via $v_{\rm el} \equiv (\lambda/\hbar NS)T_{{\rm el},z}$.

To estimate $F_{\rm el}$ and $v_{\rm el}$, we calculate spin polarization $\boldsymbol{n}(x)$ in the presence of a domain wall by use of a local gauge transformation in spin space [15], $c(x) = U(x)a(x)$, where $a(x)$ is the two-component electron operator in the rotated frame, and $U(x) \equiv \boldsymbol{m}(x)\cdot\boldsymbol{\sigma}$ is an SU(2) matrix with $\boldsymbol{m}(x) = \{\sin[\theta_0(x - X)/2]\cos\phi_0, \sin[\theta_0(x - X)/2]\sin\phi_0, \cos[\theta_0(x - X)/2]\}$. The expectation value in the presence of electric current is written in terms of the Keldysh-Green function in the rotated frame. For instance, $n_x(x) = [(1 - \cos\theta_0)\cos^2\phi_0 - 1]\tilde{n}_x + (1 - \cos\theta_0)\cos\phi_0\sin\phi_0\tilde{n}_y + \sin\theta_0\cos\phi_0\tilde{n}_z$, where $\tilde{n}_\mu(x) \equiv -i{\rm Tr}[G^<_{xx}(t,t)\sigma_\mu]$, $G^<_{x\sigma,x'\sigma'}(t,t') \equiv i\langle a^\dagger_{x',\sigma'}(t')a_{x,\sigma}(t)\rangle$, ($\sigma, \sigma' = \pm$ denotes spin) being the lesser component of the Keldysh-Green function. After a straightforward calculation, we obtain

$$F_{\rm el} = -\frac{\pi\hbar^2\Delta}{L^2}\sum_{kq\sigma} u_q^2 f_{\boldsymbol{k}\sigma}\frac{(2k + q)_x}{2m}\sigma\delta(\epsilon_{\boldsymbol{k}+q,-\sigma} - \epsilon_{\boldsymbol{k}\sigma}), \qquad (8)$$

and

$$v_{\rm el} = \frac{\hbar\lambda^2\Delta}{NSL^2}\sum_{kq\sigma} u_q^2 f_{\boldsymbol{k}\sigma}\frac{(2k + q)_x}{2m}\frac{P}{\epsilon_{\boldsymbol{k}+q,-\sigma} - \epsilon_{\boldsymbol{k}\sigma}}, \qquad (9)$$

to the lowest order in the interaction (with wall) $u_q \equiv -\int dx e^{-iqx}\nabla_x\theta_0(x) = \pi/[\cosh(\pi\lambda q/2)]$. The distribution function $f_{\boldsymbol{k}\sigma}$ specifies the current-carrying nonequilibrium state, and P means taking the principal value. As is physically expected, $F_{\rm el}$ is proportional to the reflection probability of the electron and, hence, to the wall resistivity, as well as to the charge current. In fact, by adopting the linear-response form, $f_{\boldsymbol{k}\sigma} \simeq f^0(\epsilon_{\boldsymbol{k}\sigma}) + e\boldsymbol{E}\cdot\boldsymbol{v}\tau(\partial f^0/\partial\epsilon)$, as obtained from the Boltzmann equation (f^0: Fermi distribution function; $\boldsymbol{E}$: electric field; $\boldsymbol{v} = \hbar\boldsymbol{k}/m$; τ: transport relaxation time due to a single wall), we can write as $F_{\rm el} = enjR_{\rm w}$ in one dimension. Here n and j are the electron density and current density, respectively, and $R_{\rm w} = (h/e^2)(\pi^2/8)(\zeta^2/1 - \zeta^2)(u_+^2 + u_-^2)$ is the wall resistance [16], with $\zeta \equiv (k_{F+} - k_{F-})/(k_{F+} + k_{F-})$ and $u_\pm \equiv u_{k_{F+}\pm k_{F-}}$. More generally, one can prove rigorously the relation [17,18]

$$F_{\rm el} = eN_{\rm e}\rho_{\rm w}j = enR_{\rm w}IA, \qquad (10)$$

using the Kubo formula, where $\rho_{\rm w} \equiv R_{\rm w}A/L$ is the resistivity due to a wall [19], $I \equiv jA$, and $N_{\rm e} \equiv nLA$ is the total electron number.

Equations (4) and (5) with (9) and (10) constitute a main framework of the present Letter. We next go on to studying them in the two limiting cases: adiabatic wall and abrupt wall.

We first study the adiabatic limit, which is of interest for metallic nanowires, where $\lambda \gg k_F^{-1}$. In this limit, we take $u_q^2 \to \frac{4\pi}{\lambda}\delta(q)$ and by noting $(\epsilon_{k+q,-\sigma} - \epsilon_{k\sigma})_{q=0} = 2\sigma\Delta \neq 0$, we immediately see from Eq. (8) that $F_{el} = 0$, whereas

$$v_{el} = \frac{\lambda\hbar}{NS}\frac{1}{L}\sum_{k\sigma}\sigma\frac{k_x}{m}f_{k\sigma} = \frac{1}{2S}\frac{a^3}{e}j_s \tag{11}$$

remains finite. The spin transfer in this adiabatic limit is thus proportional to spin current flowing in the bulk (away from the wall), $j_s \equiv \frac{e\hbar}{mV}\sum_k k_x(f_{k+} - f_{k-})$ ($V \equiv LA$ being the system volume). In reality, the spin current is controlled only by controlling charge current. In the linear-response regime, it is proportional to the charge current j as $j_s = \eta j$, η being a material constant. This parameter can be written as $\eta = \sum_\alpha(\sigma_+^\alpha - \sigma_-^\alpha)/\sum_\alpha(\sigma_+^\alpha + \sigma_-^\alpha)$ for a wire or bulk transport, and $\eta = \sum_\alpha(N_+^\alpha - N_-^\alpha)/\sum_\alpha(N_+^\alpha + N_-^\alpha)$ for a nanocontact and a tunnel junction, where $\sigma_\pm^\alpha$ and $N_\pm^\alpha$ are band (α) and spin ($\pm$) resolved electrical conductivity and density of states at the Fermi energy, respectively, of a homogeneous ferromagnet. Experiments indicate that η is of the order of unity in both bulk transport [20,21] and tunnel junctions (~ 0.5 [22]).

As seen from Eq. (15) below, the speed of the stream motion of the wall is roughly given by v_{el} (except in the vicinity of the threshold j^{cr}). For a lattice constant $a \sim 1.5$ Å and current density $j = 1.2 \times 10^{12}$ [A/m^2] [6], we have $a^3 j/e \sim 250$ [m/s]. This speed is expected for strongly spin-polarized materials ($\eta \sim 1$) including transition metals, but is 2 orders of magnitude larger than the observed value ~ 3 [m/s] [6]. This discrepancy may be due to dissipation of angular momentum by spin-wave emission, which is now under investigation [17].

Let us study the wall motion in the absence of pinning, $F_{pin} = 0$, by solving the equations of motion, (4) and (5) in the adiabatic case ($F_{el} = 0$). The solution with the initial condition $X = \phi_0 = 0$ at $t = 0$ is obtained as

$$\kappa\cot\left(\frac{\alpha}{\lambda}X\right) = \sqrt{1-\kappa^2}\coth(\gamma t) + 1 \qquad (|\kappa| < 1) \tag{12}$$

$$= \sqrt{\kappa^2 - 1}\cot(\gamma t) + 1 \qquad (|\kappa| > 1), \tag{13}$$

where $\kappa \equiv 2\hbar v_{el}/(SK_\perp\lambda)$ and $\gamma = [\alpha/(1 + \alpha^2)](SK_\perp/2\hbar)\times$ $\sqrt{1-\kappa^2}$. For $|v_{el}| < v^{cr} \equiv SK_\perp\lambda/2\hbar$ (i.e., $|\kappa| < 1$), $\cot(\alpha X/\lambda)$ remains finite as $t \to \infty$, and the wall is not driven to a stream motion but just displaced by $\Delta X = \frac{\lambda}{2\alpha}\sin^{-1}\kappa$. In this case, the transferred spin is absorbed by ϕ_0 and "dissipated" through $K_\perp$, as seen from Eq. (5), and is not used for the translational motion of the wall ($\dot{X}$); the wall is apparently "pinned" by the transverse anisotropy. Thus, even without pinning force, the current cannot drive the wall if the associated spin current is smaller than the critical value [23]

$$j_s^{cr(1)} = \frac{eS^2}{a^3\hbar}K_\perp\lambda. \tag{14}$$

Above this threshold, $j_s > j_s^{cr(1)}$ ($|\kappa| > 1$), this process with $K_\perp$ cannot support the transferred spin and the wall begins a stream motion. The wall velocity after "depinning" is an oscillating function of time around the average value (Fig. 1)

$$\langle\dot{X}\rangle = \frac{1}{1+\alpha^2}\frac{1}{2S}\frac{a^3}{e}\sqrt{j_s^2 - (j_s^{cr(1)})^2}, \tag{15}$$

which is similar to the Walker's solution for the field-driven case [1,24]. (The bracket $\langle\cdots\rangle$ means time average.) The asymptotic behavior $\langle\dot{X}\rangle \propto j_s$ for $j_s \gg j_s^{cr(1)}$ is governed by the angular momentum conservation (with constant dissipation rate).

We now introduce a pinning potential V_{pin} and study the "true" depinning of the wall by the spin-transfer effect in the adiabatic limit. Since spin transfer acts as a force on ϕ_0, the depinning can be better formulated in terms of ϕ_0. We consider a quadratic pinning potential with a range ξ, $V_{pin} = (NV_0/\xi^2)(X^2 - \xi^2)\theta(\xi - |X|)$, where $\theta(x)$ is the Heaviside step function. Then the equation for ϕ_0 reads $(1+\alpha^2)\ddot{\phi}_0 = -\alpha\dot{\phi}_0(\nu + \mu\cos 2\phi_0) - \nu[(\mu/2)\sin 2\phi_0 + (v_{el}/\lambda)]$, where $\mu \equiv SK_\perp/\hbar$ and $\nu \equiv 2V_0\lambda^2/\xi^2\hbar S$. This equation describes the motion of a classical particle in a tilted washboard potential $\tilde{V}$ with (modified) friction. For $v_{el} > v^{cr}(= \frac{\mu\lambda}{2})$, local minima disappear in $\tilde{V}$ and ϕ_0 is then "depinned." Then the above equation indicates that ϕ_0 starts to drift with average velocity $\langle\dot{\phi}_0\rangle = -v_{el}/(\alpha\lambda)$ (with oscillating components neglected). The displacement of $X(t)$ inside the pinning potential is then obtained from Eqs. (4) and (5) as $X \simeq (v_{el}/\nu\alpha) \equiv X_{max}$. The depinning of the wall occurs when $X_{max} > \xi$, which defines another critical current, $j_s^{cr(2)}$. Thus, the critical spin current j_s^{cr} will be given by $j_s^{cr(1)}$ defined above if the pinning is weak ($V_0 \lesssim K_\perp/\alpha$), while it is given by

$$j_s^{cr(2)} \equiv \frac{4e}{a^3\hbar}\alpha V_0\lambda^2/\xi \tag{16}$$

if the pinning is strong ($V_0 \gtrsim K_\perp/\alpha$). Since α is usually believed to be small [9], we expect that the critical current is mostly determined by $K_\perp$. This seems to be consistent with the observations that the critical current is

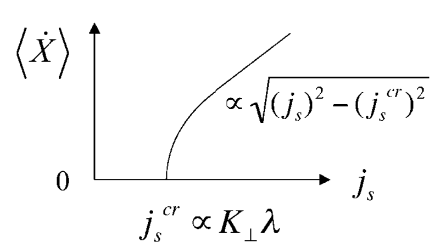

FIG. 1. Time-averaged wall velocity as a function of spin current, j_s, in the weak pinning case ($V_0 \lesssim K_\perp/\alpha$).

larger for a thinner film [6,9] and does not depend much on pinning [25]. It would be interesting to carry out measurements on a wire with small $K_\perp$.

Let us go on to the opposite limit of an abrupt wall, $\lambda \to 0$. As seen from Eq. (9), the spin-transfer effect vanishes. The pinning-depinning transition is thus determined by the competition between $F_{\rm el}$ and $F_{\rm pin}$, giving the critical current density

$$j^{\rm cr} = \frac{NV_0}{\xi e N_{\rm e} \rho_{\rm w}} = \frac{2V_0\lambda}{ena^3\xi R_{\rm w} A}. \tag{17}$$

The average wall velocity after depinning is obtained as $\langle \dot{X} \rangle = (\lambda^2 N_{\rm e} e/\hbar\alpha NS)\rho_{\rm w} j$. This velocity vanishes in the limit $\lambda \to 0$ due to the divergence of the wall mass $M_{\rm w} = \hbar^2 N/K_\perp \lambda^2$.

For metallic nanocontacts, where $\xi \sim \lambda \sim a$ and $na^3 \sim 1$, experiments indicate that the wall resistance can be of the order of $h/e^2 = 26\ {\rm k}\Omega$ [26]. Thus $j^{\rm cr} \sim (5 \times 10^{10} \times B_c[{\rm T}])\ [{\rm A/m^2}]$, where $B_c = V_0 \lambda/\mu_{\rm B}\xi S$ is the depinning field ($\mu_{\rm B}$ is Bohr magneton). $B_c \sim 10^{-3}$ [T] (like in Ref. [26]) corresponds to $j^{\rm cr} \sim 5 \times 10^7\ [{\rm A/m^2}]$.

In conclusion, we have developed a theory of domain wall dynamics including the effect of electric current. The current is shown to have two effects: spin transfer and momentum transfer, as pointed out by Berger. For an adiabatic (thick) wall, where the spin-transfer effect due to spin current is dominant, there is a threshold spin current $j_{\rm s}^{\rm cr} \sim (e\lambda/a^3\hbar)\max\{K_\perp, \alpha V_0 \frac{\lambda}{\xi}\}$ below which the wall cannot be driven. This threshold is finite even in the absence of pinning potential. The wall motion is hence not affected by the uncontrollable pinning arising from sample roughness for weak pinning ($V_0 \lesssim K_\perp/\alpha$). In turn, wall motion would be easily controlled by the sample shape through the demagnetization field and thus $K_\perp$. The wall velocity after depinning is obtained as $\langle \dot{X} \rangle \propto \sqrt{(j_{\rm s})^2 - (j_{\rm s}^{\rm cr})^2}$. In contrast, an abrupt (thin) wall is driven by the momentum-transfer effect due to charge current, i.e., by reflecting electrons. In this case, the depinning current is given in terms of wall resistivity $\rho_{\rm w}$ as $j^{\rm cr} \propto V_0/\rho_{\rm w}$.

The two limiting cases considered above are both realistic. Most metallic wires fabricated by lithography are in the adiabatic limit, as is obvious from the very small value of wall resistivity [27]. In contrast, a very thin wall is expected to be formed in metallic magnetic nanocontacts with a large magnetoresistance [26]. A system of recent interest is magnetic semiconductors [28], where the Fermi wavelength is much longer than in metallic systems. As suggested by the large magnetoresistance observed recently [29], magnetic semiconductors would be suitable for precise measurement in the thin wall limit.

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