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Memoir on the Solar Heat, on the Radiating and Absorbing Powers of the Atmospheric Air, and on the Temperature of Space. By M. Pouillet, *Member of the Royal Academy of Sciences of Paris, Professor of Natural Philosophy in the Faculty of Sciences, &c.*

[From the *Comptes Rendus des Séances de l'Académie des Sciences*, July the 9th, 1838.]

THE object of this memoir is—the quantity of solar heat which falls perpendicularly, in a given time, on a given surface:—the proportion of this heat which is absorbed by the atmosphere in the vertical passage:—the law of absorption for different obliquities:—the total quantity of heat which the earth receives from the sun in the course of a year:—the total quantity of heat which is emitted at each instant by the whole surface of the sun:—the elements which must be known in order to ascertain whether the mass of the sun cools gradually from century to century, or whether there is a cause destined to reproduce the quantities of heat which escape incessantly from it:—the elements which would allow its temperature to be determined:—the absolute quantity of heat emitted by a body whose surface, temperature and radiating power are known:—the laws of cooling of a body which loses its heat without receiving any:—the general conditions of equilibrium of temperature of a body protected by a diathermanous covering analogous to the atmosphere:—the cause of the cooling of the high regions of the air:—the law of that cooling:—the temperature of space:—the temperature which would be observable everywhere on the surface of the earth if the sun's action was not felt:—the elevation of temperature which results from the solar heat:—the relation of the quantities of heat which the earth receives from the sun, and from space or all the other celestial bodies.

It is difficult to give a brief account of the whole of these researches: I must therefore be excused both for the length of this abstract and the conciseness with which many of the propositions are presented. I regret to be unable here to go more fully into the subject, and especially that I cannot review the

labours of those who have preceded me, particularly those of M. de Laplace, M. Fourier and M. Poisson.

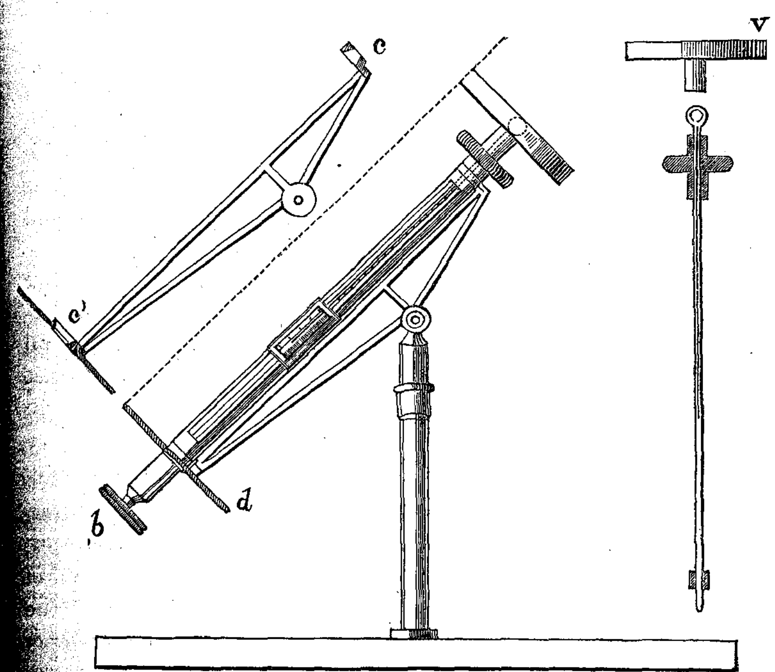
1. I have attempted to determine the quantity of solar heat by three different processes:—

(1.) By means of the apparatus which is described in the two first editions of my *Elémens de Physique et de Météorologie.*

(2.) By means of a direct pyrheliometer.

(3.) By means of a pyrheliometer with a lens.

The direct pyrheliometer is represented in the annexed figure.



The vessel V is very thin, of silver or plated metal: one decimetre in diameter, and 14 or 15 millimetres high; it contains about 100 grammes of water. The stopper which fixes the thermometer to the vessel is fitted to a metal tube which is supported at its extremities by two collets *c*, *c'*, in which it plays freely; so that on turning the button *b*, the whole apparatus turns round the axis of the thermometer, and the water of the vessel is incessantly agitated in order that the temperature may be very uniform in its whole mass. The circle d, which receives the shadow of the vessel, serves to adjust the apparatus. The surface of the vessel which receives the solar action is carefully blackened with lamp-black.

The experiment is made in the following manner:—The water in the vessel being nearly of the surrounding temperature, the pyrheliometer is held in the shade, but very near the place where it is to receive the sun: it is placed so that it looks towards the same extent of sky, and there, for four minutes, its warming or its cooling is noted from minute to minute; during the following minute it is placed behind a screen, and then adjusted so that on removing the screen at the end of the minute, which will be the fifth, the solar rays strike it perpendicularly. Then, during five minutes, under the action of the sun, its warming, which becomes very rapid, is observed from minute to minute, and care is taken to keep the water incessantly agitated; at the end of the fifth minute the screen is replaced, the apparatus withdrawn into its first position, and for five minutes more its cooling is observed.

Let R be the warming which it undergoes during the five minutes of the solar action, r and r' the coolings which it underwent during the five minutes which preceded that action and during the five minutes which followed it, it is easy to see that the elevation of temperature t produced by the heat of the sun is

$$t = \mathrm{R}\frac{(r + r')}{2}.$$

Let d be the diameter of the vessel expressed in centimetres, p the weight of the water which it contains expressed in grammes, p' the weight of the vessel itself and of the immersed portion of the thermometer, this weight being reduced to what it would be for a specific heat equal to unity, we see that the elevation of temperature observed t, corresponds with a quantity of heat

$$t\,(p + p').$$

This heat having fallen in five minutes on a surface $\frac{\pi d^2}{4}$, each unity of surface has received

$$\frac{4\,(p + p')}{\pi d^2}\,t \text{ during the five minutes,}$$

and

$$\frac{4\,(p + p')}{5\,\pi d^2}\,t \text{ during one minute.}$$

In my apparatus this quantity of heat received in one minute by each square centimetre is

$$0{\cdot}2624\,t.$$

The pyrheliometer with a lens consists of a lens of 24 to 25 centimetres diameter, having a focal distance of 60 to 70 centimetres, in whose focus is placed a silver or plated vessel containing about 600 grammes of water; the form of the vessel and the arrangement of the lens are so combined that for all heights of the sun the rays fall perpendicularly on the lens and on the face of the vessel which is to receive them at the focus and to absorb them.

The experiments are performed as with the preceding apparatus, and the quantities of heat which fall in one minute on each square centimetre are determined by an analogous formula; only there is an additional correction to be made for the quantity of heat which the lens absorbs, and that correction is made by the comparison of the results obtained with the lens and with the direct apparatus. Among the lenses which I have tried, that which absorbed the least nevertheless absorbed one-eighth of the incident heat.

It is necessary to employ the pyrheliometer with a lens when the experiments cannot be made in a calm air; the wind, when it is not strong, has but small influence in cooling within five minutes a mass of water of more than 600 grammes, which is raised only four or five degrees above the surrounding temperature, so that the correction is always very small.

2. The following table contains five series of experiments, which will give a sufficient idea of the variations of the direct pyrheliometer. The elevations of temperature observed are in the third column. We shall show hereafter how the numbers of the second and fourth columns have been obtained.

Hours of Observation.	Atmospheric Densities, or ε.	Observed rises of Temperature.	Calculated rises of Temperature.	Differences.
Observations of the 28th of June, 1837.				
7h 30′ morn.	1·860	3°·80	3°·69	+0·11
10h 30′ morn.	1·164	4·00	4·62	−0·62
Noon	1.107	4·70	4·70	0·
1h	1·132	4·65	4·67	−0·02
2	1·216	4·60	4·54	+0·06
3	1·370	......	4·32	0·
4	1·648	4·00	3·95	+0·05
5	2·151	......	3·36	
6	3·165	2·40	2·42	−0·02
Observations of the 27th of July, 1837.				
Noon	1·147	4·90	4·90	0·
1h	1·174	4·85	4·86	−0·01
2	1·266	4·75	4·74	+0·01
3	1·444	4·50	4·51	−0·01
4	1·764	4·10	4·13	−0·03
5	2·174	3·50	3·49	+0·01
6	3·702	3·35	3·42	−0·07
Observations of the 22nd of September, 1837.				
Noon	1·507	4·60	4·60	0·
1h	1·559	4·50	4·54	−0·04
2	1·723	4·30	4·36	−0 06
3	2·102	4·00	3·97	+0·03
4	2·898	3·10	3·24	−0·14
5	4·992	......	1·91	
Observations of the 4th of May, 1838.				
Noon	1·191	4·80	4·80	0·
1h	1·223	4·70	4·76	−0·06
2	1·325	4·60	4·62	−0·02
3	1·529	4·30	4·36	−0·06
4	1·912	3·90	3·92	−0·02
5	2·603	3·20	3·22	−0·02
6	4·311	1·95	1·94	+0·01
Observations of the 11th of May, 1838.				
11h	1·193	5·05	5·06	−0·01
12	1·164	5·10	5·10	0·
1	1·193	5·05	5·06	−0·01
2	1·288	4·85	4·95	−0·10
3	1·473	4·70	4·73	−0·03
4	1·812	4·20	4·37	−0·17
5	2·465	3·65	3·67	−0·02
6	3·943	2·70	2·64	+0·06

3. After having obtained, in the course of several years, a larg number of series analogous to the preceding, I endeavoured t discover a law which might represent with sufficient exactne

all the results of the observations. To effect this, I first calculated the atmospheric thicknesses which the solar rays had to traverse in each experiment; these thicknesses ε are given by the formula

$$\varepsilon = \sqrt{2rh + h^2 + r^2\cos^2 z} - r\cos z;$$

r is the mean radius of the earth, h the height of the atmosphere, z the zenithal distance of the sun; I adopted

$$h = 1,\ r = 80.$$

With respect to the zenithal distance z, instead of determining it each time by observation of the sun's altitude, I preferred to take the precise hour of the middle of the experiment, and to deduce the value of z from the formula

$$\cos z = \sin v \sin d + \cos v \cos d \cos \mathrm{H}.$$

v is the latitude of the spot where the observation is taken, d the declination of the sun at noon, H the horary angle of the sun corresponding to the hour of the experiment.

By means of these two formulæ I calculated the atmospheric thicknesses given in the second column of the preceding table.

4. By comparing the elevations of temperatures observed on the pyrheliometer and the corresponding atmospheric thicknesses, I saw that the results might be very well represented by the formula

$$t = \mathrm{A}p^{\varepsilon},$$

A and p being two constants. Moreover, by determining these two constants by two observations of each series, we always obtain the same value of A for all the series, and different values of p in passing from one series to the other. Thus A is a fixed constant, independent of the state of the atmosphere, and p a constant which is fixed only for the same day, and which varies from one day to another, according as the serenity of the sky is more or less perfect. A is therefore, in the formula, the *solar constant*, or that which contains, as its essential element, the constant calorific power of the sun, whilst p is the atmospheric constant, or that which contains, as its essential element, the power of variable transmission, which the atmosphere possesses to allow portions of the incident solar heat, more or less great, to arrive at the surface of the earth.

The experiments give for A the value

$$6^{\circ}\!\cdot\!72,$$

and for p the values contained in the following table:—

Dates of the Series.	Values of p.	Values of $1-p$.
June 28	0·7244	0·2756
July 27	0·7585	0·2415
September 22 . . .	0·7780	0·2220
May 4	0·7556	0·2444
May 11	0·7888	0·2112
Winter solstice . . .	0·7488	0·2512

By means of these values of A and of p, and of the formula

$$t = \mathrm{A}p^{\varepsilon},$$

I calculated the results contained in the fourth column of the preceding table; it is seen with what accuracy all the numbers which had been given by observation are thus reproduced, even when the observation corresponds to atmospheric thicknesses, which are quadrupled by the effect of obliquity. Thus in the experiments of the 4th of May, the solar rays had to traverse an atmospheric thickness of 24 leagues at noon, and 86 leagues at six o'clock in the evening, and yet the number calculated is still found perfectly to agree with the number observed. It is understood, however, that it is only when the weather is quite settled that the formula can be applied with accuracy to an entire day with the same value of p; if there occur any sudden changes in the state of the atmosphere, the values of p immediately experience a greater or less alteration: I have succeeded in convincing myself of this by a multitude of experiments corresponding to all the seasons of the year. It may even be assumed that in certain spots, especially in mountainous countries and near the sea-coast, the values of p undergo every day periodical variations, corresponding to the diffusion and to the condensation of the vapours.

5. If in the preceding formula we suppose $p = 1$, or $\varepsilon = 0$, we find

$$t = 6^{\circ}{\cdot}72;$$

that is to say, that the pyrheliometer would assume an elevation of 6°·72 if the atmosphere could transmit wholly all the solar heat without absorbing any of it, or if the apparatus could be transported to the limits of the atmosphere to receive there without any loss all the heat which comes to us from the sun. This value of t multiplied by 0·2624, gives

$$1{\cdot}7633.$$

This then is the quantity of heat which the sun gives in 1′ on a square centimetre, at the limits of the atmosphere, and which

it would equally give at the surface of the earth, if the atmospheric air did not absorb any of the incident rays.

6. The preceding values of p indicate the proportions of solar heat which have been transmitted in the different days to which they correspond, and the values of $1-p$ indicate, on the contrary, the different proportions of solar heat which have been absorbed at the same periods. These values, however, correspond to $\varepsilon = 1$, that is to say, they indicate the proportions of solar heat which would have been transmitted and absorbed at those places where the sun was in the zenith, supposing there the same atmospheric state as at Paris at the moment of the experiment. It results that in the vertical passage the atmosphere absorbs at least $\frac{21}{100}$ of the incident heat, and at most $\frac{27}{100}$, without the sky ceasing to be serene. I must however add, that on the 28th of June, to which corresponds the absorption of $\frac{27}{100}$, a light white veil was visible on the vault of the sky. Moreover other observations, the series of which could not be completed, indicated only an absorption of $\frac{18}{100}$; we may thus say that the atmospheric absorption is comprised between 18 and 24 or 25 hundredths, without its being possible to distinguish in the sky vapours which disturb its transparence.

7. By means of this datum, and of the law according to which the transmitted heat diminishes in proportion as the obliquity increases, we may calculate the proportion of incident heat which each instant reaches the illuminated hemisphere of the earth, and that which is absorbed in the corresponding half of the atmosphere. This calculation depends on an integral of the form

$$c\int \frac{p^{\varepsilon}\, d\varepsilon}{\varepsilon^{2}},$$

which cannot be obtained exactly; but by various methods of approximation, it is easy to recognise that for $p = 0{\cdot}75$ the proportion which reaches the ground is comprised between 0·5 and 0·6, and consequently the proportion absorbed by the atmosphere is itself comprised between 0·5 and 0·4; but very near 0·4.

Thus, when the atmosphere has all the appearance of perfect serenity, it still absorbs nearly the half of the total quantity of heat which the sun emits towards the earth, and it is the other half only of that heat which falls upon the surface of the ground, and which is there variously distributed, according as it has traversed the atmosphere with greater or less obliquities.

8. Having ascertained the quantity of heat which the sun sends to the earth during 1′, by its perpendicular action upon a square centimetre, it is easy to determine the total quantity of heat which the entire globe of the earth and the atmosphere receive each minute. In fact, that quantity of heat is what would fall upon the circle of illumination if the hemisphere of the earth, which is at the same time illuminated and heated by the sun, were removed. Now the surface of this circle of illumination being πR^2, the total quantity of heat which it receives is

$$1{\cdot}7633 \, . \, \pi R^2.$$

If this heat were uniformly dispersed over all points of the earth, each square centimetre would receive for its share only

$$\frac{1{\cdot}7633 \, . \, \pi R^2}{4 \, \pi R^2} \text{ or } 0{\cdot}4408.$$

It is easy to see, according to this, that in the course of a year the total quantity of heat received by the earth from the sun is the same as if, during that interval, there should enter, by each square centimetre of the surface which limits the atmosphere,

231675 unities.

By transforming this quantity of heat into a quantity of melted ice, we obtain the following result:—

If the total quantity of heat which the earth receives from the sun in the course of a year were uniformly dispersed over all points of the globe, and if it were there employed, without any loss, in dissolving ice, it would be capable of dissolving a stratum of ice which would envelope the whole earth, and which would have a thickness of $30^{m}{\cdot}89$,

or nearly 31 metres; such is the simplest expression of the total quantity of heat which the earth receives every year from the sun.

9. The same fundamental datum enables us to solve another question, which will perhaps appear more bold, and the solution of which is nevertheless quite as simple. It enables us to find the total quantity of heat which escapes from the entire globe of the sun in a given time, without supposing anything, except that all the equal portions of the globe of the sun emit equal quantities of heat; this appears hitherto confirmed by experiment, since the different aspects which the sun presents to us by the effect of its rotation do not appear to have any marked influence upon the terrestrial temperatures.

Let us consider the centre of the sun as the centre of a spherical inclosure, the radius of which is equal to the mean distance of the earth to the sun: it is evident, that upon this vast inclosure every square centimetre receives in 1′ from the sun precisely as much heat as the square centimetre of the earth, that is to say 1·7633; consequently, the total quantity of heat which it receives is equal to its entire surface, expressed in centimetres and multiplied by 1·7633, or to

$$1{\cdot}7633 \, . \, 4 \, \pi \, D^2.$$

This incident heat is nothing else than the total sum of the quantities of heat emitted in all directions by the entire globe of the sun, that is to say by a surface $4 \, \pi R^2$, R being the radius of the sun. Thus each square centimetre emits for its share

$$1{\cdot}7633 \, . \, \frac{D^2}{R^2} \text{ or } \frac{1{\cdot}7633}{\sin^2 \omega};$$

ω being the visual demi-angle at which the sun is seen from the earth, that is to say $15' - 40''$; which gives 84888: thus each square centimetre of the solar surface emits in 1′

84888 unities of heat.

By transforming this heat into a quantity of melted ice, we obtain the following result:—

If the total quantity of heat emitted by the sun were exclusively employed in dissolving a stratum of ice applied upon the globe of the sun, and enveloping it on every side, that quantity of heat would be capable of dissolving in 1′ a stratum $11^{m}{\cdot}80$ thick, and in one day a stratum of 16992^{m}, or $4\frac{1}{4}$ leagues. This determination, as will have been seen, does not rest upon any hypothesis; it is independent of the peculiar nature of the sun, of the matter of which it is composed, of its radiating power, of its temperature and of its specific heat; it is simply the immediate consequence of the principles best established in relation to radiating heat, and of the number which we have attained by experiment.

10. This same subject may give rise to a multitude of questions; we will further examine the two following, less with a view of solving them than of indicating the number and the nature of the unknown elements on which their solution depends.

The first question is to ascertain whether there is, in the very substance of the sun itself, a source destined to produce heat, to repair in some manner, by chemical, electrical or other actions, the losses of calorific rays which take place every instant; or whether, if these losses recurring incessantly without any reparation, there results, from century to century, a progressive diminution of temperature in which the globe of the earth must participate.

According to what we have just seen, each square centimetre of the sun loses in 1′ a quantity of heat $v = 84888$ unities; in m number of minutes it loses $m\,v$; and the entire sun loses

$$4\,\pi\,R^2 . m\,v.$$

Now, if we suppose that the mass of the sun has a perfect conductibility for heat, so that its temperature is the same throughout,—if we represent its mean density by d and its mean capacity for heat by c, it is evident that, to be lowered 1°, the entire mass of the sun must lose a quantity of heat expressed by

$$\frac{4}{3} . R^3\,\pi\,d\,c,$$

since in m number of minutes it loses $4\,\pi\,R^2\,m\,v$; it follows that during this time it decreases by a number of degrees given by the relation

$$\frac{3\,v\,m}{R . d . c}.$$

The radius of the sun expressed in centimetres is 70 billions; the mean density d of the sun, as compared with water, is 1·4; it is deduced from the mean density of the earth 5·48, from the mass of the sun, which is 355 thousand times that of the earth, and from its volume, which is 1384 thousand times that of the earth.

Taking, moreover, for m the number of minutes which correspond to a year, namely 526000, and substituting for v its value 84888, this relation becomes

$$\frac{4}{3\,c}.$$

This is the number of degrees which the mass of the sun must cool each year, on the hypothesis of a perfect conductibility; if to this first hypothesis we add a second with relation to specific heat,—if, for example, we suppose that it is 133 times the specific heat of water, we find that the entire mass of the sun would then cool

$\frac{1}{100}$ of a degree per annum,
or 1 degree in a century,
or 100 degrees for 10 thousand years.

Thus the solution of the question before us now depends only on two elements which, without doubt, will remain for ever unknown to us, namely, the conductibility of the mass of the sun, and its capacity for heat; and we have just seen how, by means of these two elements, should we arrive at a knowledge of them, the question might be resolved in an exact manner. With regard to the hypotheses I have made respecting them, their only object is to show the extent of the uncertainties to which science is subjected on this point.

11. With the same object we shall proceed to examine another question, which however has the advantage over the preceding one of being more accessible to science, namely, whether the temperature of the sun may have some analogy with the temperatures which we are able to produce by chemical or electrical actions.

We shall see, in the following article, that the total quantity of heat emitted in 1′ by a square centimetre of surface is always expressed by

$$1{\cdot}146 . f . a^t,$$

f being the emissive power of that surface, t its temperature, and a the number 1·0077 determined with great exactness by MM. Dulong and Petit.

We have found elsewhere that for the sun this quantity of heat is 84888. Then

$$\text{for } f = 1, \quad t = 1461,$$
$$\text{for } f = \tfrac{1}{10}, \quad t = 1761.$$

Thus the temperature of the sun depends on the law of radiation of heat and of the emissive power of the sun or of its atmosphere. In a former memoir* I have described an air pyrometer, by means of which I determined all the high temperatures up to the fusion of iron; I have since verified the fact that the law of radiation applies to temperatures which exceed 1000°; these experiments will soon enable me to ascertain whether the law in question extends in fact to temperatures of 1400° or 1500°; but we may even now regard such extension as very probable. With respect to the emissive power of the sun, it is unknown, but we cannot suppose it greater than unity. It follows therefore that the temperature of the sun is at least 1461°, that is to say nearly that of the melting point of iron, and that

* *Comptes Rendus de l'Académie des Sciences*, t. iii. p. 782.

this temperature might be 1761° if the emissive power of the sun were analogous to that of polished metals. These numbers do not differ much from those which I had determined by other principles and by other means of observation in my memoir of 1822.

12. Starting from the laws of cooling *in vacuo*, discovered by MM. Dulong and Petit, and developing a particular point of view which those able men of science had already indicated in their work, I was led to this general theorem.

The absolute quantity of heat e which is emitted in the unity of time by the unity of surface of any body, the temperature of which is $t + \theta$, and the emissive power of which is f, is always expressed by the relation

$$e = \mathrm{B} . f . a^{t+\theta},$$

B being an invariable constant, which depends solely on the zero of the scale and on the unities of time and of surface; its value is 1·146, taking the minute and the square centimetre for unities.

To demonstrate this general law of the emission of heat, let us consider a spherical body submitted to cooling or to the equilibrium of temperature in the centre of an inclosure also spherical; let us suppose that the body and the inclosure have, both of them, a maximum emissive power, in order to avoid the effects of reflexion; let us designate by e the quantity of heat which is emitted by the unity of surface of the inclosure, and let us suppose that the equilibrium of temperature is established; the total quantity of heat lost by the body in the unity of time is

$$e\, s,$$

s being the surface or $4 \pi r^2$.

If we represent by e'' the portion of this heat which is received and absorbed by the unity of surface of the inclosure, we shall have

$$e''\, s'$$

for the total quantity of heat received by the inclosure, s' being its entire surface or $4 \pi r'^2$.

Now the quantity lost by the body being equal to that which is received by the inclosure, we shall first have

$$e\, s = e''\, s',$$

whence

$$e'' = e . \frac{s}{s'} = e . \frac{r^2}{r'^2} = e \sin^2 \omega,$$

ω being the demi-angle at which the body is seen from a point of the inclosure.

If we now consider what the body receives from the inclosure, we easily perceive that it is a certain fraction b of the quantity of total heat e' which is emitted by each element, and consequently that it receives from the entire inclosure a quantity of heat expressed by

$$b\, e'\, s'.$$

Since the equilibrium is established, the quantity which the body receives is equal to the quantity which it loses, which gives

$$b\, e'\, s' = e\, s,$$

whence

$$b\, e' = e \frac{s}{s'} = e \frac{r^2}{r'^2} = e \sin^2 \omega = e'';$$

that is to say, the entire body receives, from each element of the inclosure, a quantity of heat which is precisely equal to that which it imparts to it.

But at equilibrium, the temperatures of the body and of the inclosure being equal, the quantities e and e' must also be equal, because their emissive power is the same; therefore

$$b = \sin^2 \omega.$$

Thus, whilst each element of the inclosure emits in all directions a certain quantity of heat e', the body receives from that element only

$$e' \sin^2 \omega.$$

It is clear, moreover, that if, the temperature of the inclosure remaining constant, that of the body changes, the body will not less receive from the inclosure this quantity $e' \sin^2 \omega$ which it received at equilibrium, e' being always the total quantity of heat emitted in all directions by the unity of surface of the inclosure.

Now if it be true that the absolute quantity of heat emitted in the unity of time by the unity of surface is expressed by a function of the form

$$e = \mathrm{B} f a^{t+\theta}$$

there results, for the total quantity $e\, s$ of heat lost by the body,

$$e\, s = s . \mathrm{B} . f a^{t+\theta};$$

at the same time, the inclosure having the same emissive power f and the temperature θ, we shall thus have

$$e' = \mathrm{B} . f\, a^{\theta},$$

and for the total quantity of heat emitted by the inclosure,

$$s' e' = s' . \mathrm{B} . f a^{\theta}.$$

As the body receives only a portion, $\sin^2 \omega$, of this heat, its real and definitive loss will therefore be

$$s e - s' e' \sin^2 \omega = s \mathrm{B} f . a^{t+\theta} - s' \sin^2 \omega . \mathrm{B} . f a^{\theta},$$

or on account of $s' \sin^2 \omega = s$,

$$s . \mathrm{B} . f (a^{t+\theta} - a^{\theta}).$$

Such is the quantity of heat lost by the body.

If now we represent its weight by p and its specific heat by c, it is evident that for one unity of heat which it loses, its temperature is only lowered a number of degrees denoted by

$$\frac{1}{c p}.$$

Consequently, whilst it loses a number of unities of heat expressed by

$$s \mathrm{B} . f (a^{t+\theta} - a^{\theta}),$$

it loses in temperature only a number of degrees denoted by

$$\frac{s . \mathrm{B} . f}{c p} (a^{t+\theta} - a^{\theta}):$$

this is, properly speaking, its velocity of cooling.

To make this formula coincide with that of MM. Dulong and Petit, it is sufficient to suppose

$$m = \frac{s . \mathrm{B} . f}{c p},$$

and it would moreover be necessary that the constant should be null, if it had been added to the value of e, as is easy to be seen supposing the body only to be polished; this demonstrates the exactness of the general relation

$$e = \mathrm{B} f a^{t+\theta}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

and it shows, at the same time, the elementary composition of the coefficient m, the numerical value of which has been given by the experiments of cooling; thus the magnitude of this coefficient is in direct ratio to the surface of the body and its radiating power, and in inverse ratio to the mass of the body and its capacity for heat.

With respect to the value of the constant B, it may be deduced from the preceding relation, at least very approximately,

since the coefficient m has been determined with great care by MM. Dulong and Petit, and found equal to 2·037 for a thermometer with glazed surface which was spherical, filled with mercury, and which was 6 centimetres in diameter.

Assuming therefore

$$m = 2{\cdot}037,$$
$$c = 0{\cdot}033,$$
$$\frac{s}{p} = \frac{1}{13{\cdot}65},$$
$$f = 0{\cdot}8,$$

we find

$$\mathrm{B} = 1{\cdot}146.$$

This result cannot be perfectly exact, both because the value of f is a little hypothetical, and because the true dimensions of the thermometer in question being wholly useless for the researches on cooling, MM. Dulong and Petit have only indicated them in a general manner: it is however certain that the error cannot be considerable, and we shall adopt the value of B as sufficiently near*.

13. We may, indeed, demonstrate directly, in another way, that the values of the coefficient m are undoubtedly in a direct ratio to the surface and the emissive power of the bodies subjected to cooling, and in an inverse ratio to the weight of these bodies and their capacity for heat.

In fact, admitting that the velocity of cooling in absolute cold is expressed as in the formula of MM. Dulong and Petit, by the relation

$$v = m a^{t},$$

we obtain by integration the following formula:

$$x = \frac{1}{m\, l' a}\left(\frac{a^{\mathrm{T}-t} - 1}{a^{\mathrm{T}}}\right), \quad . \quad . \quad . \quad . \quad (3.)$$

in which T represents the initial temperature of the body, and x the number of minutes which elapse whilst the body falls from the initial temperature T to any temperature t.

Consequently, for the body to be lowered 1° is required a time expressed by

$$x = \frac{1}{m\, l' a}(a - 1)\, a^{-\mathrm{T}}.$$

Now, if we represent the surface of the body by s, its weight by p, and its specific heat by c, it is evident that in falling 1° it

* See Note 1, p. 85.

loses a quantity of heat $p\,c$, and as it loses it by one surface s, each unity of surface loses

$$\frac{p\,c}{s};$$

but since the body requires a time x to fall 1°, it follows that in a time 1 it falls

$$\frac{1^\circ}{x}.$$

Thus in the unity of time, the unity of surface loses a quantity of heat expressed by

$$\frac{p\,c}{s}\cdot\frac{m\,l'\,a}{a-1}\cdot a^{\mathrm{T}}.$$

For another body which should have the same initial temperature T, the loss would be

$$\frac{p'\,c'}{s'}\cdot m'\cdot\frac{l'\,a}{a-1}\,a^{\mathrm{T}}.$$

As these losses must be proportional to the radiating powers f and f' of the two bodies, we should have

$$\frac{m}{m'}=\frac{s\,.\,f\,.\,p'\,c'}{s'\,.\,f'\,.\,p\,c},$$

that is to say, the coefficients m and m' are in fact in a direct ratio to the surfaces and the radiating powers, and in an inverse ratio to the masses and the capacities.

14. The formulæ (2) and (3) contain the laws of cooling in absolute cold; they may be employed to solve a great number of questions.

The first shows, for example, that under the equator, where the temperature of the earth is on an average 30°, each square centimetre loses in unities of heat,

1·44 in 1',
1037·00 in 12 hours;

whence it follows, that in a column of water 10 metres in depth, there would in twelve hours be only a lowering of 1°, if by its upper surface that column lost its heat in absolute cold without receiving compensation for what it loses, either by its free surface or by its sides.

The second shows that in absolute cold the thermometer of MM. Dulong and Petit would take

34'·14 to fall from 100° to 0,
74'·66 to fall from 0 to −100°;

but this small globe was only 6 centimetres in diameter, and if we make the same calculations for a similar body having, for example, the dimensions of the earth, we find that in absolute cold this globe would take

13,640 years to fall from 100° to 0,
29,830 years to fall from 0 to −100°.

These examples may show that there has perhaps been some exaggeration in the ideas which have been hitherto entertained of absolute cold and the phænomena which would be manifested on the surface of the earth if the temperature of space were excessively reduced below the zero of our thermometers; they show at the same time that the essential laws of heat are established upon such fixed principles that sudden changes of temperature are not less impossible in the system of the world than the sudden changes resulting from mechanical actions.

15. The theorem relative to the emission of heat enables us to determine the conditions of equilibrium of temperature of the atmosphere; for this purpose we shall proceed to consider in a general manner the conditions of equilibrium of temperature of a globe protected by any diathermanous covering and suspended with its envelope in the middle of a spherical inclosure.

Let us designate by s, s'', s' the surfaces of the globe, of the envelope, and of the inclosure; by e, e'', e' the quantities of heat emitted in the unity of time by each of the unities of surface of s, s'', s'; we designate by b the absorbing power which the diathermanous envelope exerts upon the heat emitted by the globe, and by b' the absorbing power which it exerts upon the heat emitted by the inclosure.

The globe emits in the unity of time a quantity of heat $e\,s$; one portion $b\,e\,s$ is absorbed by the envelope, and one portion $(1-b)\,e\,s$ traverses the envelope to reach the inclosure.

The inclosure emits a total quantity of heat $e'\,s'$; one part $e'\,s'\sin^2\omega$ falls upon the diathermanous envelope, designating by ω the demi-angle at which the inclosure is placed with regard to the envelope; this latter absorbs a portion of it, $e'\,s'\,b'\sin^2\omega$, and it lets pass a portion, $e'\,s'\,(1-b')\sin^2\omega$.

The envelope emits a quantity of heat $e''\,s''$ toward the globe, and an equal quantity of heat $e''\,s''$ toward the inclosure.

The sum of the quantities of heat which the envelope loses is equal to the sum of the quantities of heat which it receives, and this gives a first equation,

$$2\, e'' s'' = b\, e\, s + b'\, e'\, s' \sin^2 \omega.$$

We obtain in the same manner for the globe and for the inclo sure, two other equations which result from the equality between the quantities of heat received and lost, namely,

$$e\, s = e'' s'' + (1 - b')\, e'\, s' \sin^2 \omega,$$
$$e'\, s' \sin^2 \omega = e'' s'' + (1 - b)\, e\, s.$$

It is easy to see that these three equations are reducible to two, because the first is a consequence of the two last, and might be deduced from them.

If we now suppose the radius of the envelope to be sensibly equal to the radius of the globe, as is nearly the case with the atmosphere around the earth, the equations become

$$e = e'' + (1 - b')\, e',$$
$$e' = e'' + (1 - b)\, e,$$

and this leads to the three following relations:

$$\frac{e}{e'} = \frac{2 - b'}{2 - b},$$
$$\frac{e}{e''} = \frac{2 - b'}{b + b' - b\, b'},$$
$$\frac{e'}{e''} = \frac{2 - b}{b + b' - b\, b'}.$$

If, now, we designate by t, t'', t' the temperatures of the glob of the envelope and of the inclosure; by f, f'', f' their emissi powers, we shall have, in virtue of the principle above esta blished, the three other equations:

$$e = \mathrm{B} \,.\, a^{t},$$
$$e' = \mathrm{B} \,.\, a^{t'},$$
$$e'' = \mathrm{B} \,.\, f''\, a^{t''},$$

on the supposition we have made, to simplify the matter, th the globe and the inclosure have maximum emissive powers.

These equations, combined with the preceding ones, give—

$$a^{t - t'} = \frac{2 - b'}{2 - b},$$
$$a^{t - t''} = f'' \cdot \frac{2 - b'}{b + b' - b\, b'},$$
$$a^{t' - t''} = f'' \frac{2 - b}{b + b' - b\, b'}.$$

Such are the general relations which give in all possible cases the differences of temperature required by the conditions of equilibrium between the globe and the inclosure, the globe and the envelope, the inclosure and the envelope. We see that these differences depend essentially upon the relative values of b and of b', that is to say, on the absorbing powers which the diathermanous envelope exercises upon the heat of the globe and upon that of the inclosure.

If we first suppose that these absorbing powers are equal, that is to say that $b = b'$, it results that

$$t = t',$$
$$a^{t - t''} = \frac{f''}{b},$$
$$a^{t' - t''} = \frac{f''}{b}.$$

Thus all diathermanous envelopes which exert equal absorbing powers on the rays of heat of the globe and the inclosure, do not prevent the globe and the inclosure having for the equilibrium exactly the same temperature as if the diathermanous inclosure did not exist, and *vice versâ*.

With regard to the temperature of the diathermanous envelope itself, we see that it can only be equal to that of the globe and of the inclosure, under the condition that $f'' = b$, that is to say, that the emissive power of that envelope is equal to its absorbing power, which, in fact, is the case with rock-salt and air, as I have ascertained by experiment.

But when these conditions are not fulfilled, when the diathermanous envelope exerts unequal absorbing powers on the heat of the inclosure and on that of the globe, the principle of the equality of temperature ceases to be true, and immediately there are then manifested, contrary to the ordinary laws of equilibrium, differences more or less considerable between the temperatures of the globe, the inclosure and the envelope. The following table contains some of the results which are obtained in discussing the formulæ, after having attributed different values to b' and to b.

Values		Excess of temperature		
of b'.	of b.	of the globe over the inclosure $t-t'$.	of the globe over the envelope $t-t''$.	of the inclosure over the envelope $t'-t''$.
0·3	0·7	35·0	53·5	18·5
0·3	0·8	45·5	59·5	14·0
0·3	0·9	57·0	65·0	8·0
0·4	0·8	38·0	49·0	11·0
0·4	0·9	49·0	56·0	7·0
0·5	0·9	41·0	46·0	5·0
0·5	0·95	46·5	49·5	3·0
0·0	0·9	78·0	91·0	13·0
0.0	0·1	91·0	91·0	0·0

It results, for example, that if the diathermanous envelope absorbs only three-tenths of the heat of the inclosure and eight-tenths of that of the globe, the temperature of the globe then exceeds that of the inclosure by 45°·5, and that of the envelope by 59°·5, which last is thus 14° below the temperature of the in-closure itself.

There is, however, a limit to the accumulation of heat upon the globe and to the cooling of the envelope, and that limit is 91°,

This effect of diathermanous envelopes is very remarkable, and it becomes perhaps still more striking when we ascend to the temperatures themselves instead of stopping at their simple dif-ference, since the preceding examples then lead to this result, that if an inclosure has its sides kept throughout at the tempe-rature of melting ice, a globe suspended in the centre of this in-closure, having no other heat but what it receives from it, may nevertheless, under certain conditions, be raised to the tempe-rature of 40° to 50° above zero, that is to say, to a temperature considerably higher than that of the torrid zone, and maintain this excess of temperature without ever cooling, on condition of no longer being in equilibrium of temperature, and consequently of being instantly heated again by the rays of the heat of the in-closure. For this phænomenon, it is sufficient for the globe to be protected by a diathermanous envelope, possessing the double property of absorbing only half the heat emitted by the surface of the inclosure, and of absorbing on the contrary about nine-tenths of the heat emitted by the surface of the globe.

Lastly, to complete this consequence, with relation to the en-velope itself, which is the sole cause of this effect, we must also add, that this envelope included between an inclosure at zer

and a globe at 45° or 50° would be found to have only a mean temperature lower by some degrees than zero, its lower strata being warmer than the inclosure, and its upper strata much colder, following a certain law of decrease which may be calcu-lated when the proper data are obtained.

Our present remarks (supposing the inclosure to be at the temperature of melting ice, or rather supposing that the heat which reaches the globe is uniformly distributed and equivalent in quantity to that which would proceed from such an inclosure possessing a maximum emissive power) apply, under the same conditions, to an inclosure of any temperature, provided that this temperature does not vary from the degrees of heat or cold to which the law of cooling may extend.

Such, in general, are the effects produced by diathermanous envelopes from the inequality of the absorbing actions which they may exercise upon the different rays of heat which traverse them; with respect to the cause of these unequal absorptions, Delaroche has demonstrated, on one side, that it depends on the sources of heat themselves, and consequently on the peculiar nature of the calorific rays; and M. Melloni has shown, on the other hand, that it depends also, under certain relations, on the nature of the diathermanous substances.

16. Hitherto it has been admitted that two athermanous sur-faces of the same temperature emit identical rays of heat, or at least rays of heat which experience always equal absorptions in traversing the same media; but it is perhaps not impossible to arrive at the discovery, in this respect, of some differences which depend either upon the diversity of the emissive powers, or on the very nature of the bodies.

This is an essential point, to which the researches of M. Melloni have, without doubt, not failed to call the attention of natural philosophers. If these rays, emanating from sources of equal temperature, resist all tests, if they preserve their identity in traversing the same diathermanous media, it will be impossible to obtain, in the experiments of the laboratory, any accumula-tion of heat by the interposition of diathermanous envelopes; since in that case the absorbing powers of these envelopes would necessarily be the same upon the rays of the inclosure and on those of the globe or of the interior thermometer.

This impossibility, however, could not affect the consequences which we shall proceed to deduce from the formulæ with relation to the effects which the atmosphere produces either upon the heat of the sun, or upon the heat of the other heavenly bodies, which is in general designated under the name of *heat of space*, or *planetary heat*.

With regard to the solar heat no doubt exists: we know that in traversing diathermanous substances it is less absorbed than the heat which is derived from different terrestrial sources, the temperature of which is not very high. It is true that we have been able to make the experiment only upon solid or liquid diathermanous screens; but we regard it as certain that the atmospheric stratum acts in the manner of screens of this kind, and that consequently it exercises a greater absorption upon the terrestrial than upon the solar rays; we must also add, that this difference of action does not result, as is sometimes said, from the solar heat being luminous and the terrestrial heat obscure; for, up to the present day, all that is known with regard to this subject leads us to think that there is neither warm light nor luminous heat: the rays of heat and of light may derive their origin from the same source, be emitted at the same time and coexist in the same pencil of rays, but they preserve a distinctive character, since, on the one hand, they may be separated the one from the other, and on the other hand, there is no instance of a ray of heat which has been transformed into a ray of light, nor of a ray of light, properly speaking, which has been transformed into a ray of heat. The inequality of absorption in question depends therefore on peculiar properties which the rays of heat assume when they are emitted by sources at a temperature more or less high, and these properties are only maintained, or perhaps developed more, when the temperature of the sources is sufficiently raised for them to emit, like the sun, light at the same time as heat.

With respect to the heat of space, there is another distinction to be made: it must be considered in relation to its quantity and in relation to its nature.

Considered in relation to its quantity, it is measured, as all other heat, by the effects which it produces; that is to say, by the quantity of ice which it can melt, or by the elevation of temperature which it would impart to a given quantity of water. It is upon this principle that M. Fourier has first shown that it was necessary to take account of the heat of space in order to explain the phænomena of the terrestrial temperatures; and it

is upon this principle also that he has indicated, in a general manner, that the temperature of space ought to be very little inferior to the temperature of the poles of the earth, and about 50° or 60° below zero; expressing by this valuation nothing else than that the total heat which reaches the earth from all the celestial bodies, excepting the sun, is equivalent in quantity to that which would be emitted on the globe of the earth by an inclosure at a maximum emissive power, the sides of which should be kept at the temperature of 50° or 60° below that of melting ice. The essential point in this manner of regarding the phænomena is the possibility of substituting for the whole of the celestial bodies a fictitious inclosure, or an athermanous surface kept throughout at a certain temperature; it remains to examine whether there are experiments by which this temperature itself can be determined, and with what degree of approximation we may hope to obtain it.

Considered in relation to its nature, the heat of space gives rise to a multitude of questions which it would be useless to treat of here; we shall limit ourselves therefore to some observations inherent to our subject. We remark, first, that if the fictitious inclosure which has just been spoken of can, when a proper temperature is assigned to it, represent strictly, or very nearly, the heat of space, it can only represent it for its quantity; and it will never represent it for its nature, since the heat of space possesses essentially properties due to its origin, which it could not derive, without doubt, from a source the temperature of which should be lower than melting ice. We instantly see that from this there result conditions which it is impossible for us to reproduce in our experiments, namely, a heat which is by its quantity as if it emanated from a cold source, and by its nature as if it emanated from a hot source. To explain this kind of contradiction it suffices to admit that any line, starting from the earth and prolonged indefinitely in space, will not essentially encounter a body which can transmit heat to the earth; or, in other words, it is sufficient to admit that the planetary inclosure, in reality, is not for us an inclosure becoming continuous by the assemblage of numberless bodies which are dispersed in the depths of space at distances continually increasing; then, in fact, there will be points or small portions of the celestial vault which would transmit heat to us, and other portions doubtless larger which would not transmit heat to us, because the corresponding lines are prolonged indefinitely in space.

We thus understand that the heat of space may be assimilated to the solar heat in its nature and origin, if not in its quantity, and that the atmosphere consequently exerts upon it the same absorption. This being established, the general conditions of equilibrium of the diathermanous envelopes which we have discussed above find here their direct application: it is sufficient to admit that the globe which we have supposed of any given dimensions, is the globe of the earth; that the inclosure is that which represents the unknown temperature of space; and, lastly, that the diathermanous envelope is nothing else than the atmosphere supposed at first without clouds, and possessing the property of absorbing only in the perpendicular direction about 20 or 25 hundredth parts of incident heat, as we have found by the experiments on the solar heat related above. As the absorbing action which the atmosphere exerts upon the rays emitted by the earth is necessarily greater, it results that all the consequences at which we have arrived apply to the equilibrium of the terrestrial temperatures.

In consequence, the phænomena which occur without the action of the sun and without the effects of the interior heat of the globe are the following:—

1. The temperature of the surface of the earth is considerably higher than the temperature of space:

2. The mean temperature of the atmosphere is necessarily lower than the temperature of space, and still more than the temperature of the earth itself:

3. The decrease of the temperature in the atmosphere is not at all due to the periodical action of the sun, nor to the ascending and descending currents which that action may determine near the surface of the earth; it will even take place when the sun would not heat either the earth or the atmosphere, because it is one of the conditions of equilibrium of diathermanous envelopes, and its real cause lies in the unequal absorbing actions which the atmosphere exerts upon the rays of heat derived from space and upon those which are emitted all round the globe by the surface of the soil or by that of the seas.

M. Fourier is, I think, the first who has had the idea of regarding the unequal absorption of the atmosphere as exercising

an influence on the temperatures of the soil. He had been led to this by the beautiful experiments made by De Saussure, in 1774, on some elevated summits of the Alps and in the adjacent plains, with a view to compare the relative intensities of solar heat. On that occasion* M. Fourier states in a precise manner one of the principles which have served me to establish the equations of equilibrium; only that he appeared to apply it merely to the solar action, supposing that this periodical action is the principal cause of the decrease of temperature of the atmosphere.

On another side, M. Poisson, in his last work, has already shown that the upper strata of the atmosphere must necessarily be at a much lower temperature than the temperature of space: he has deduced this result, on one hand, from the numbers at which he has arrived to express the temperature of space, and on the other, from the mechanical conditions of equilibrium, which could not be fulfilled at the limits of the atmosphere if the air did not there experience a degree of cold sufficient to make it lose all its elasticity. This consequence, which might appear extraordinary when presented only as a mechanical necessity, may perhaps now appear, if not more certain, at least more natural, since it results also from the laws of radiant heat, and since it is by this explained and referred to its real origin.

17. If we now return to the conditions of equilibrium of diathermanous envelopes to examine the causes which may have influence upon their double absorbing power, we shall remark that the specific heat of the substance of these envelopes cannot change without the absorbing powers changing also in a certain relation. In fact, if around the globe we substitute for a given envelope another envelope of the same mass and the same matter, which differs from it only in its capacity for heat, it is extremely probable that the effects will be different, that these two envelopes will not take the same temperature, and that they will not determine equal accumulations of heat on the globe, even supposing that the relative values of the two absorbing powers remain the same in each of them.

This simple remark, joined to some other considerations which cannot be developed here, has led me to admit that the absorbing powers of the same elastic fluid, considered as a diathermanous substance, are proportional to its mass and to its capacity for

* *Annales de Chimie*, tome xxvii. p. 155.

heat. Thus, in dividing the atmosphere, for example, into 100 concentric strata of the same mass, the individual absorbing powers of any two strata will be proportional to the different specific heat of these two strata. Near the surface of the earth, where the pressure is great and the capacity small, the proportion of heat absorbed will be consequently less than near the limits of the atmosphere, where the pressure is feeble and the capacity considerable; we see that at the same time the inferior stratum occupies a vertical height much smaller than that of the upper stratum. This consideration modifies, as we have stated, the quantities of solar heat which reach the summits of the high mountains, and it leads to a general expression for these quantities of heat, in which it remains to substitute the barometric pressures and the corresponding specific heats. It is thus that the absorption, which we have found and verified by experiment, may extend to the different heights to which it is possible to ascend in order to make observations there analogous to those which we have made at Paris.

Lastly, this same principle, and those which have been developed above, lead us to express, in a simple manner, the total quantity of radiant heat which is emitted in a given time by the unity of surface of any atmospheric strata whatever. This quantity of heat depends, in fact, only on the peculiar temperature of that stratum which we shall represent by t, on its capacity for heat c, and on its mass m, then on the number $B=1{\cdot}146$ which is the constant of the radiation; and lastly, on an unknown constant k which depends on the nature of the elastic fluid; its value is then

$$B\,k\,m\,c\,a^{t}.$$

For another stratum of the same mass, situated at a greater height, the temperature of which would be t' and its capacity c', the total quantity of heat lost in the same time would be

$$B\,k\,m\,c'\,a^{t'}.$$

This being established, let us consider the state of the atmosphere under the equator, admitting that the sky there has been long unclouded, and that the equilibrium of temperature is then established throughout the height of the atmospheric column; then, the mean temperature of each day being nearly constant upon the soil, and constant also in each of the strata of air at whatever height, the soil and the different strata of the atmo-

sphere must lose every day all the heat that they receive. Now the quantity of heat received by one of the lower strata, for example, depends upon the absorbing power which is peculiar to it, and on the incident heat which reaches it, both from the earth below, and also from the sun and from space above. It is the same with respect to one of the upper strata: only it is evident that this will receive from the sun and from space much more incident heat than the lower stratum, since this heat is weakened more and more in proportion as it penetrates into deeper strata; it is also evident that the lower stratum in turn will receive in compensation much more terrestrial heat than the upper stratum, because the terrestrial heat is weakened by the same cause in proportion as it penetrates into more elevated strata. The relation of these received quantities, or rather of the quantities received and absorbed by any two strata, may be calculated approximately, and we find that it does not vary much from unity, provided at least that we do not arrive at strata very near the limits of the atmosphere: if we assume it equal to unity, this signifies that two strata of air, one upper and the other lower, very near to or very distant from one another, absorb each day equal quantities of heat; but since they both lose all that they receive, it results very evidently that they lose in the same time equal quantities of heat. Thus, we should have

$$B\,k\,m\,c\,a^{t} = B\,k\,m\,c'\,a^{t'},$$

whence

$$t - t' = \frac{1}{l\,a}\,.\,l\,.\,\frac{c'}{c}.$$

This result, which expresses in so simple a manner the law of the decrease of the temperature of the air for the equatorial region, and which seems to extend nearly to the limits of the atmosphere, requires to be verified by experiment, as much at least as these verifications are possible.

Now we know, by the researches of M. de Laplace and M. Poisson, that the capacities of elastic fluids for heat are connected with the pressures which these fluids support by a relation of the form

$$\frac{c'}{c} = \left(\frac{p}{p'}\right)^{1-\frac{1}{k}},$$

which becomes for dry air

$$\frac{c'}{c} = \left(\frac{p}{p'}\right)^{\frac{3}{11}},$$

and we equally know that this formula has been verified by very accurate experiments of MM. Gay-Lussac and Welter, which extend for the pressures from 1460 mill. to 144 mill., and for the temperatures from 40° above zero to 20° below.

Thus we may already calculate the capacities of the different strata of air up to four-fifths of the height of the atmosphere: it would be, however, interesting to continue the experiments of M. Gay-Lussac, and to extend them, if it be possible, preserving the same precision, down to temperatures of 60° or 80° below zero, —a temperature which we can now obtain by means of the apparatus of M. Thilorier*.

However, if we admit provisionally that the formula of M. Poisson actually extends to a pressure of $\frac{1}{100}$ of atmosphere, we find that the temperature of the corresponding stratum at that pressure would be lower by 163° than the mean temperature of the stratum nearest the soil, and, as the latter is 27° above zero, the other would be 136° below zero.

Calculating the temperatures of the 100 strata corresponding to each of the 100ths of the atmospheric pressure, and taking the mean of them, we obtain approximately what we may call the mean temperature of the atmospheric column, because it is in fact in virtue of this temperature that the entire column emits radiant heat: the calculation gives for this mean − 8°.

There is still another verification possible. We know that the barometric formula is exact up to a considerable height, and that it establishes a relation between the vertical distance of the two strata and the corresponding pressures. This relation is approximately

$$z = 18393 \,.\, l \,.\left(\frac{p}{p'}\right);$$

on combining it with the preceding, we arrive at this result—

$$t - t' = \frac{z}{224{\cdot}8},$$

that is to say, that the difference of the temperatures of two strata is 1° for every 225 metres within the extent to which the barometric formula can be applied.

We know that the experiments of M. de Humboldt give 200; this difference of one-eighth depends doubtless on many causes, and particularly on the fact that the formula which connects the capacities with the pressures can only be employed for dry air.

* See my experiments on this subject, *Comptes Rendus*, t. iv. p. 513.

whilst the air is in general very humid under the equator, on account even of its temperature.

18. A thermometer which is exposed on the ground to the nocturnal radiation receives heat from two different sources, namely from space and from the atmosphere. The heat of space being submitted to absorption like the solar heat during its atmospheric passage, only three- or four-tenths of it can in general reach the thermometer, at least supposing that the experiments are not made on high mountains. With respect to the heat emitted by the atmosphere itself in the course of the night, it is the effect of the individual radiation of all the concentric strata which we can imagine from the level of the sea up to the limits of the atmosphere, and it depends consequently on the distribution of the temperatures throughout the whole height of the atmospheric column; we may add, that its influence is much more considerable than has hitherto been supposed. For the rest, whatever be the relation of the intensities of these two causes, it is evident that we may conceive a single cause capable of producing an effect equal to that which results from their simultaneous action; or, in other terms, we may suppress in thought the heat of space and that of the atmosphere, and conceive an inclosure, of a maximum emissive power, the temperature of which is such that it imparts to the thermometer and to the ground precisely as much heat as they receive at once from the atmosphere and from space; this is the unknown temperature of that *zenithal inclosure* which I term the *zenithal temperature.*

The object of this manner of viewing the phænomena is not to represent the peculiar and perhaps unequal actions which the thermometer experiences in such or such a direction, but only to represent with exactness the definitive and total action to which it is submitted, so that its depression below the ambient temperature is the same with the zenithal inclosure as with the atmosphere and space united. It is under this condition that we are now permitted to assign to the zenithal inclosure a uniform temperature in all the portions of its extent. In short, it is evident that the zenithal temperature is necessarily variable at each instant for the same point of the surface of the earth, and still more variable for one point than another, because it is composed of a fixed element, which is the temperature of space, and of an element incessantly changing, which is the temperature of the different atmospheric strata.

The advantage there may be in resolving thus the problem will be better comprehended when we shall have shown the new relations which thence result between the unknown quantities which we seek to determine. Let us represent by z the zenithal temperature and preserve the same designation for the other quantities, namely,—

t' for the temperature of space;
t'' for the mean temperature of the atmospheric column;
b for the absorbing power which the atmosphere exerts upon the terrestrial heat; and
b' for the absorbing power which the atmosphere exerts upon the celestial heat.

This being established, let us consider,—

1. That during the unity of time the zenithal inclosure emits by the unity of surface a quantity of heat,

$$B\,a^{z};$$

B being the same constant 1·146 of which we have previously spoken; there is no coefficient relative to the radiating power, because we must suppose it equal to unity.

2. That the atmosphere emits likewise a quantity of heat

$$B\,b\,a^{t''},$$

because its emissive power is equal to its absorbing power, which we have represented by b.

3. Lastly, that space emits a quantity of heat

$$B\,a^{t'},$$

but that there is only one portion of it $(1-b')$ which directly traverses the atmosphere to reach the ground, whence it follows that with relation to the thermometer which rests upon the ground it is as if space had an emissive power $1-b'$, and as if it transmitted only a quantity of heat

$$(1-b')\,B\,a^{t'}.$$

Since the zenithal inclosure replaces the atmosphere and space, the quantity of heat which it emits with relation to the thermometer must be exactly equal to the sum of the quantities of heat emitted by the atmosphere and space.

We have then

$$B\,a^{z} = B\,b\,a^{t''} + (1-b')\,B\,a^{t'},$$

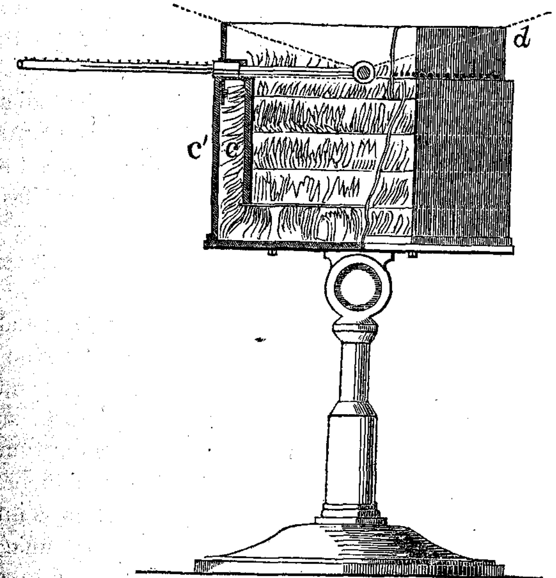
$$a^{z} = b\,a^{t''} + (1-b')\,a^{t'}. \quad . \quad . \quad . \quad . \quad (4)$$

Such is the general relation which connects incessantly the zenithal temperature with the temperature of space, with the mean

and variable temperature of the column of air, and with the two unequal absorbing powers of the atmosphere.

19. Let us now attempt to indicate how it is possible to observe the zenithal temperature each instant of the night, nearly as we observe the temperature of the air.

I have employed two methods for this purpose: one which rests on the employment of mirrors, and the other on the employment of a novel instrument which I call *actinometer*; we know that this name is already applied to a very important invention of Sir J. Herschel, and it seems to me happily chosen by that illustrious astronomer to designate all kinds of apparatus the object of which is to measure the effects of radiation, whatever be the principle of their construction.



It will be sufficient to indicate here the second method: I shall only remark with regard to the first, that the cooling which is observed at the focus of a mirror whose axis is directed towards the zenith, does not depend upon the concentration of the rays, as has been hitherto supposed; a simple plate of polished metal, or rather an expanded cone, produces nearly the same effect, so that I have found it possible to substitute for the mirrors reflectors of this kind which are much more convenient. However, with the reflectors, as with the mirrors, the experiments are delicate and the formulæ very complicated; they contain the real temperature of the air, and the relation of the cooling resulting from its contact to that resulting from the radiation—two data upon which it is impossible not to have some uncertainty.

The actinometer is represented in the above figure: it is composed of four rings of two decimetres diameter, furnished with swans'-down, and resting one upon another, so that the down may not experience any compression: the swans' skin itself forms the foundation of the circle of each of these rings. This system is inclosed in a first cylinder of silver plate c, enveloped also in swans' skin, and contained in a larger cylinder c'. A thermometer rests on the centre of the upper down: the margin d is of such a height that the thermometer can only face two-thirds of the hemisphere of the sky; this margin is perforated with holes at the level of the down, in order that the cold air may flow regularly *.

This apparatus is during the night exposed to the radiation of the sky, and its thermometer and an adjoining thermometer freely suspended in the air at four feet from the ground are observed hourly; the zenithal temperature is deduced from the difference of these temperatures, or from the lowering of the actinometer; but for this the apparatus must have received a graduation, which we proceed to indicate.

20. If the actinometer had an indefinite surface, and were maintained at a constant temperature *in vacuo* under an hemispherical inclosure, it would evidently take the temperature of the inclosure; on the contrary, with its real form, facing only two-thirds of the hemisphere, and enveloped in a stratum of air which warms it again, it must always remain at a higher temperature than that of the inclosure. The graduation is intended to determine how much it is heated, so that it is sufficient to know its temperature and that of the ambient air to deduce thence the temperature of the inclosure, with which it is in interchange of radiant heat. It is obvious that there must exist a simple relation between the temperature of the inclosure and the lowering of the actinometer. To discover this relation I composed an artificial sky with a zinc vessel a metre in diameter, supported at a height of two metres by three slender columns; this vessel, the bottom of which was blackened, was filled with a refrigerating mixture at $-20°$, and the actinometer was placed vertically beneath at such distances that the centre

* See Note 2, p. 89.

thermometer faced successively extents corresponding to one quarter hemisphere, one-third hemisphere, and two-thirds hemisphere; in each position I awaited the equilibrium of temperature, and noted at the same time the temperature of the ambient air and that of the apparatus. Analogous experiments, repeated at the temperature of melting ice and at other intermediate temperatures, have led me to the following result:—If from the ambient temperature nine-fourths of the lowering of the actinometer be taken away, we always find the temperature of the artificial sky. This result evidently applies to the celestial vault, or rather to the zenithal inclosure; consequently, if we observe during the night the temperature t of the ambient air, and the lowering d of the actinometer, we shall thence deduce the zenithal temperature by the formula

$$z = t - 9\frac{d}{4},$$

which is the result of the graduation.

21. Further on will be found a table containing some of the series of experiments which have been made during very beautiful nights, and in calm weather, to determine the zenithal temperature. These experiments confirm the fact that the zenithal temperature is lowered during the night, nearly as the temperature of the ambient air; this progressive lowering, from the setting to the rising of the sun, is an essential fact which leads immediately to an important consequence.

In fact, we have seen that the zenithal temperature is expressed by the sum of two terms; one depending on the mean temperature of the atmospheric column, which is variable, and the other depending on the temperature of space, which is fixed. Now since the zenithal temperature experiences, in a single night, considerable variations, this is an evident proof that the fixed term which enters into its expression has only a very small value with relation to the variable term, and consequently that, in the nocturnal radiation, the heat of space is very small in relation to the heat which is derived from the radiation of the atmosphere.

This consequence can scarcely be reconciled with the opinions which attribute to space a temperature the value of which would not be lowered beneath zero a very great number of degrees; but it is perfectly reconcileable with the known facts which already might have furnished indications in this direction, had they all been analysed with the attention which they merit.

The numerous results of Mr. Wells, of Mr. Daniell, and of all the other natural philosophers who have made experiments on nocturnal radiation, not only prove that a thermometer exposed on the ground during the night, in an open place, is cooled 6°, 7°, or even 8° below the ambient temperature; they also prove that this phænomenon is reproduced, almost with the same intensity, in the coldest months of the year, that is to say in January and February, when the temperature of the air has fallen many degrees below zero. Thus Wilson observed a difference of nearly 9° between the temperature of the air and that of the surface of the snow; Scoresby and Captain Parry have observed analogous depressions in the Polar regions, when the temperature of the air was more than 20° below zero.

If we now consider that the heating power which the stratum of air exerts by its contact with the thermometer of the ground which is colder than it, is nearly the same, whether it be at 10° above zero or 10° below, it results that the cooling power which maintains this thermometer at −18° in the second case, has also the same energy as the cooling power which maintains it at +2° in the first case; and, as this cooling power depends on the temperature of space, it follows also that the temperature of space is much lower than −18°; for if it were only −30° or −40°, the thermometer which is at −18° whilst the air is at −10° would be too near for the heat of space to be able to keep it at the same depression below the air as the thermometer which is at +2° whilst the air is at 10°. What has perhaps hitherto prevented our arriving at this conclusion, is that in the explanations which have been given of the nocturnal radiation, a peculiar cooling power has been generally attributed to the upper strata of the atmosphere, which are known to be very cold, forgetting in some measure that, cold as they are, it is nevertheless heat which they transmit, and that this heat is added to that of space to augment its effects.

The results which I have obtained by means of the actinometer are therefore in accordance with all known facts; it was perhaps essential to make this remark, in order to show that, if the consequences at which we shall arrive are in some points contrary to received opinions, this results from the nature of things rather than from the inaccuracy of the experiments.

22. Considering the equation (4.) as an equation of condition which must always be satisfied for every value of the zenith temperature given by experiment, I have been able to determine

the limits for the temperature of space; but the phænomena which are manifested in the equatorial regions, and which there prevail in a constant manner throughout the year, lead to another fundamental equation, from which we may derive the temperature of space without having recourse to the mean temperature of the atmospheric column.

In fact, in the equatorial zone, the surface of the earth, including the atmosphere which covers it, may be considered as a cylinder, the tropical circles of which would form the two bases, and the half of which is always illuminated by the sun. This cylinder receives each instant all the heat which falls upon the rectangle of its projection, the surface of which is $2rh$; it receives therefore each minute a quantity of heat

$$1{\cdot}7633 \,.\, 2rh.$$

But this quantity of heat being distributed over the whole lateral surface of the cylinder or upon an extent $2\pi rh$, it is evident that each unity would only receive for its share

$$\frac{1{\cdot}7633}{\pi} = 0{\cdot}56.$$

Such is the quantity of solar heat which on an average falls daily every minute on each square centimetre of the equatorial zone.

At the same time the heat of space also exerts its action, and if we designate by t' the unknown temperature of space, it is easy to see that the quantity of heat received per minute and per square centimetre is

$$\mathrm{B}a^{t'}.$$

Consequently the sum of the quantities of heat received is

$$\mathrm{B}a^{t'} + 0{\cdot}56.$$

But the combined effects of space and of the sun may be replaced by a single inclosure, of a maximum emissive power; and if we represent by v the unknown temperature of this inclosure, capable of producing the same effects, or rather capable of transmitting the same quantity of heat, we shall have

$$\mathrm{B}a^{v} = \mathrm{B}a^{t'} + 0{\cdot}56;$$

it is true that the action of the sun is intermittent, since it ceases to be felt during the night, and during the day it is felt with different intensities at different hours; but these intermittences, producing the variations of temperature which we observe during the day and the night, do not interfere with the exactness of the preceding equation; nor do they prevent the conditions of equilibrium of diathermanous envelopes applying rigorously to the inclosure, whose unknown temperature we have just designated by v.

This temperature v must therefore be such that it may produce on the surface of the earth, between the tropics, the mean temperature of 27°·5, which results from observation. But we have seen that the excess of temperature of the globe over the inclosure is always deduced from the formula

$$a^{t-t'} = \frac{2-b'}{2-b},$$

t being the temperature of the globe, and t' the temperature of the inclosure.

Now, here, the temperature of the globe being 27°·5, and that of the inclosure v, we must therefore have

$$a^{27°\cdot5-v} = \frac{2-b'}{2-b}.$$

If we take the value of a^v which thence results, and substitute in the preceding equation, and also for B its value 1·146, we find

$$a^{t} = 1\cdot235\,\frac{2-b}{2-b'} - 0\cdot489.$$

And as the whole of the solar experiments gives $b' = 0\cdot35$, we arrive definitely at the equation

$$a^{t'} = 1\cdot008 - 0\cdot748\,.\,b, \quad . \quad . \quad . \quad . \quad (5.)$$

in which the only unknown values are the temperature of space t and the absorbing power b, which the atmosphere exercises upon the terrestrial heat.

The greatest value of b gives the lower limit of the temperature of space; and since b cannot be greater than 1, the temperature of space cannot be lower than

$$-175°.$$

For $b' = 0\cdot3$ we should find -187, and for $b_{,} = 0\cdot4$ only -1

This lower limit once being found, it is easy to ascertain the superior limit; for it corresponds to the smallest value which it is possible to attribute to b: now the experiments of zenith temperature showing that b is necessarily greater than 0·8, it results that the temperature of space is less than

$$-115°.$$

To determine now the intermediate number, comprised between these limits, which represents the true temperature of space at the present period, numerous experiments will doubtless be requisite, extending to all latitudes and all heights.

The only experiments that I have yet been able to make permit us to arrive however at a certain approximation; they give

$$-142°$$

for the temperature of space, and I do not think that this value can be far from the truth; it corresponds to $b = 0\cdot9$. Thus we see, as the definitive result of these researches, that the sun imparts to the earth a quantity of heat 1·7633 per minute and per square centimetre, that with a clear sky the atmosphere absorbs about four-tenths of this heat and of that of space; that it absorbs nine-tenths of the heat emitted by the earth, and that the temperature of space at the present period is 142° below zero. We can scarcely draw attention too strongly to the important part which the inequality of the absorbing powers of the atmospheric air exercises on the whole of the terrestrial phænomena, and consequently to the care necessary to determine them with accuracy. For this purpose doubtless other apparatus and other methods of experimenting will be soon invented, by means of which it will be possible to separate at every instant the complex influences of the radiation of space and the atmospheric radiation. If at the present time the different regions of the sky which pass successively to the zenith appear to us to transmit equal quantities of heat, it is very probable that this only results from the imperfection of our apparatus: we perceive such differences in the nature, distance, number and grouping of the stars in the depths of space, that it is impossible to admit that the portion of the heavens, incessantly changing, which is above the horizon, resembles constantly the portion that is below; and consequently it is impossible that all the hemispheres which we can conceive in the celestial vault transmit really to the earth the same quantity of heat. It is especially in the equatorial zone that we must first seek to appreciate these differences, because they must doubtless appear there greater, more regular and more easy of observation.

23. The following table contains the result of experiments made with the actinometer: the progressive depression of the zenithal temperature will be remarked; the last column of this table contains the mean temperature t'' of the atmospheric column at Paris, corresponding to each observation, and calculated by the formula (4) of the zenithal temperature in which this quantity t'' remains alone unknown.

Table of the Mean Temperatures of the Atmosphere which correspond to the observations of the Actinometer made during the months of April, May and June.

Days.	Hours.	Temper. of the air.	Temper. of the Actinom.	Differ.	Zenithal temp.	Mean temp. of the atmosphere.
From the 10th to the 11th of April.						
April 10.	7h evening.	10·2	3·9	6·3	− 4·0	−23·5
	8	9·9	3·0	6·9	− 5·6	−25·5
	9	9·6	2·2	7·4	− 7·0	−27·0
	10	9·0	1·8	7·2	− 7·2	−27·5
... 11.	5 morning.	5·0	−3·0	8·0	−13·0	−35
	5 30′	5·0	−3·0	8·0	−13·0	−35
	6	5·5	−2·3	7·8	−12·0	−34
From the 14th to the 15th of April.						
April 14.	7h evening.	8·5	0 8	7·7	− 6·0	−26
	8	7·0	−0·5	7·5	− 9·9	−30·0
	9	5·8	−1·6	7·4	−10·8	−32
	10	5 0	−2·4	7·4	−11·6	−33·5
... 15.	4 30′ morning.	1.0	−6·0	7·0	−14·7	−37·5
	5	1·0	−6·0	7·0	−14·7	−37·5
	6	1·6	−5·2	6·8	−13·7	−36·0
From the 20th to the 21st of April.						
April 20.	8h evening.	5·6	−0·8	6·4	− 8·8	−29·5
	9	4·5	−2·0	6·5	−10·1	−31·5
	10	3·6	−3·0	6·6	−11·7	−33·5
... 21.	4 30′ morning.	0 0	−7·0	7·0	−15·7	−38 5
	5	0·0	−7·0	7·0	−15·7	−38·5
	5 30′	0·1	−6·5	6·6	−14·5	−37·0
From the 5th to the 6th of May.						
May 5.	5h evening.	25·50	19·9	5·6	+12·9	− 2·0
	6	25·10	17·5	7·6	8·0	− 8·0
	7	23·10	15·0	8·1	4·9	−12·0
	8	22·9	13·9	9·0	2·6	−15·0
	9	21·5	12·5	9·0	1·4	−16·5
	10	17 5	10	7·5	0·6	−17·5
... 6.	4 morning.	12·1	5	7·1	−3·9	−23·5
	4 30′	12·1	5	7·1	−3·9	−23·5
	5	12	6	6·0	−1·5	−20·0
From the 23rd to the 24th of June.						
June 23.	7h evening.	20·0	12·0	8 0	+2·0	−16·0
	8	17·8	10·5	7·3	1·4	−16·5
	9	17·6	10·7	6 9	,,	,,
	10	16·3	9·2	7·1	0·3	−18·0
... 24.	4 morning.	11·3	5·3	6·0	−2·2	−21·0
	4 30′	11·5	5·6	5·9	−1·8	−20·5

24. It seems to me still necessary to indicate some of the most general consequences which result from these researches.

The total quantity of heat which space transmits in the course of a year to the earth and to the atmosphere is deduced from the preceding observations: it is easy to see that this quantity of heat would be capable of melting upon our globe a stratum of ice of

26 metres thickness.

We have seen that the quantity of solar heat is expressed by a stratum of ice of

31 metres.

Thus, together, the earth receives a quantity of heat represented by a stratum of ice of

57 metres,

and the heat of space concurs in this for a quantity which is five-sixths of the solar heat.

Between the tropics, the heat of space is only two-thirds of the solar heat; for the latter is there represented by a stratum of ice of

39 metres.

It will excite astonishment doubtless that space, with its temperature of −142° below 0, can impart to the earth a quantity of heat so considerable that it is nearly equal to the mean heat which we receive from the sun; these results appear, at first sight, so contrary to the opinion which we form, either of the cold of space or of the power of the sun, that we shall be perhaps disposed to regard them as inadmissible. However, we must remark that, with regard to the earth, the sun occupies only five-millionths of the celestial vault, and that it must consequently transmit 200,000 times more heat to produce the same effect.

In considering the phænomena under another point of view, we shall be led, on the contrary, to suppose that in these valuations the power of the sun is much exaggerated; for if we examine the temperatures instead of examining the quantities of heat, we arrive at this result:—

That if the sun's action were not felt upon our globe, the temperature of the surface of the ground would throughout be uniform and at

−89°.

Now, since the mean temperature of the equator is 27°·5, we must conclude that the presence of the sun augments the temperature of the equatorial zone by

$$116^{\circ}{\cdot}5.$$

In the same manner, the mean temperature of the atmospheric column would be at the equator

$$-149^{\circ}.$$

The preceding formulæ show that it is about -10°; thus, the intermittent presence of the sun increases by

$$139^{\circ}$$

the mean temperature of the entire atmosphere in the torrid zone. This effect of the sun to augment the terrestrial temperature much exceeds that which M. Poisson obtained in considering the variations of temperature at different depths beneath the surface of the ground; but it appears to me that the two methods will yield results more accordant, when we can introduce in a more direct manner, into the formulæ, the influence of the atmosphere, which is so considerable.

To extend these calculations to other regions, we must take into account the decrease of the temperature of the ground in proportion as the latitude increases; but, by approximation, it is easy to recognise that the effects of the wind concur to raise the temperature of the polar regions, by lowering more or less the temperatures of the regions comprised between the polar and tropical circles; the temperature of the equatorial zone itself appears little lowered by this cause.

The object of this extract is especially to give an idea of the theoretical principles and the experimental methods which serve as the basis of this investigation. I may be allowed to invite the attention of geometricians and natural philosophers particularly to these two points: with respect to the numbers which result from my experiments, they will have to be modified; subsequent researches, simultaneously undertaken on different points of the globe, will be necessary to give them all the precision which they require.

Note 1.

The mode of demonstration which I have used in this article permits the solution of several difficulties which are still presented by the most fundamental principles of radiant heat,—namely, the principle of the equality of temperature and of the equality of the emissive powers of two surfaces which do not reflect; the law of the cosinus; and the law of emission, considered in relation to the constant which the calculation gives, and which is added to the function of temperature.

Principle of the equality of temperature and of the equality of the emissive powers of two surfaces which have no reflective power.—It may be admitted as evident, that two equal surfaces of the same matter, both possessing a maximum emissive power, emit equal quantities of heat in the same time, when they have equal temperatures; for, all being identical, the quantities of heat emitted must be themselves identical.

Thus, for this particular case, if the globe and the inclosure have the same temperature, we have $e = e'$, the equilibrium is established, and we have at the same time $b = \sin^2 \omega$.

This result is independent of the dimensions of the globe and the inclosure: now, the same globe, at the same temperature, losing always the same quantity of heat in the same time, it results that all the inclosures, large or small, transmit to the globe the same quantity of heat; it results, consequently, that at any portion of a given inclosure, seen from the centre of the globe at a certain angle, we may always substitute a portion of another inclosure seen at the same angle; this tends to prove that the temperature of the globe is equal to that of the inclosure, not only when it is at the centre of that inclosure, but whatever be the position that it occupies.

If now the globe is not of the same substance as the inclosure, or, in a word, it ceases to be identical with it, without ceasing to have, like it, a complete absorbing power, the loss of the globe will then be $e_1 s$; the portion of heat which it receives from the inclosure and which it absorbs will always be the same, namely, $e' s' \sin^2 \omega$; and the sole condition required for the equilibrium will in like manner be, that the quantities of heat lost and absorbed may be equal, which gives again

$$e_1 s = e' s' \sin^2 \omega,$$

whence

$$e_1 = e';$$

that is to say, even in this case it is necessary for the equilibrium that the unity of the surface of the globe loses in the same time as much heat as the unity of the surface of the inclosure.

It remains to examine whether, to undergo this loss, the globe must still have the temperature of the inclosure: now, if it must have, for example, a higher temperature, this excess being independent of its dimensions, it would be necessary that it should preserve it when it had sufficient volume to fill the inclosure; whence it follows that there might be a permanent difference of temperature between two bodies which touch, or at least which are very near to one another, a thing that is contrary to all experiments.

Thus, two equal surfaces, possessing a complete absorbing power, always emit equal quantities of heat when they are at the same temperature, whatever be the difference of matter or property between them.

It is evident that, in this case, the principle of the equality of temperature, at equilibrium, cannot be demonstrated *à priori*, and that it results only from the general indications of experiment.

But the second principle, that of the equality of the emissive powers of any two surfaces of the same temperature, which do not reflect, is immediately deduced from the first; it is upon this point that some uncertainty remained*.

Law of the Cosinus.—The law of the cosinus, which appears to leave a doubt on the mind of some natural philosophers†, is also deduced in a very simple manner from the same considerations.

Since for the equilibrium we have always $e = e'$ and $b = \sin^2 \omega$, where the globe and the inclosure have complete absorbing powers, and since each unity of surface of the inclosure transmits to the globe a quantity of heat expressed by $e \sin^2 \omega$, ω being the angle of emission, that is to say, the angle of the extreme rays with the normal on the surface, it is evident that, for an angle ω' a little larger than ω, the quantity of heat would be

$$e \sin^2 \omega';$$

whence it follows that the quantity of heat emitted in the zone comprised between ω' and ω is

$$e (\sin^2 \omega' - \sin^2 \omega);$$

the surface of this zone being $2\pi (\cos^2 \omega - \cos \omega')$, the quantity of heat emitted from the corresponding unity of surface is therefore

$$\frac{e}{2\pi} (\cos \omega + \cos \omega'),$$

or

$$\frac{e}{\pi} \cos \omega,$$

supposing that the difference is very small between ω and ω'.

That is to say, if we consider the element of a radiating surface as the centre of a hemisphere whose radius is equal to unity, and if we call e the total quantity of heat emitted by this element in all directions, the quantity of heat emitted on a single element of the hemisphere, which is at an angular distance ω from the normal, is expressed by

$$\frac{e}{\pi} \cos \omega,$$

and is consequently proportional to the cosinus of the angle of emission, this angle being reckoned to begin from the normal.

It is easy, starting from this value, to re-ascend by integration to the primitive expression $e \sin^2 \omega$.

This demonstration is perhaps more elementary than that which has been given by M. Fourier*; it does not suppose any consideration relative to the temperature, and applies consequently to all temperatures; it is deduced from a single condition, which is, that we have $b = \sin^2 \omega$, and, as the quantity of heat which the globe receives from the inclosure is independent of the heating or the cooling of the globe itself, we see that it is not less strict for the case of heating or cooling than for the case of equilibrium.

Constant of Emission.—After having found the relation

$$m\, a^{\theta} + \text{constant},$$

MM. Dulong and Petit remark (page 74) that it represents the total radiation of the inclosure, and that, the origin of the temperatures being arbitrary, we may assume it so that the constant may be null, which would reduce the expression to $m\, a^{\theta}$.

But there remains some difficulty upon this point; for, in choosing the 0 of the thermometric scale so that θ may be equal to $-\infty$, the first term is null, and the constant remains, so that it could only disappear by admitting that it must have a negative value: moreover, the value of m necessarily changes with the origin of the temperatures; it diminishes in proportion as the origin descends and as the temperatures are represented by greater numbers, and reciprocally; in short, the value of m is such, that the total radiation of the inclosure appears expressed rather by a temperature than by a quantity of heat, and nevertheless it is difficult to conceive how the total radiation of the inclosure can be expressed by a temperature or by a number of degrees lost or gained.

On another side, M. Poisson†, on comparing the functions to which he has arrived and those of the law of cooling of MM. Dulong and Petit, preserves the constants in the first functions, and suppresses them in those of the law of cooling, which re-

* M. Poisson, *Théorie de la Chaleur*, p. 42.

† Ibid. p. 35

* *Annales de Chimie et de Physique*, tom. iv. p. 135.

† *Théorie de la Chaleur*, p. 42.

verts to the supposition that they are different in the two cases; but it must be remarked that all the experiments which can be made being relative to changes of heat in which these constants are destroyed, no observation can ever give their value.

There remains therefore respecting this constant of emission a theoretical difficulty the more serious, because upon it alone depends the impossibility of considering the absolute emission and of assigning a value to the real and total quantities of heat which are emitted by the radiating surfaces. It appears to me, however, that this difficulty is not incapable of solution: the constant in question is only a product of the calculation; it cannot represent any mechanical condition nor any physical phænomenon, and its value must be null in all cases.

Let us in fact suppose that e, or the total quantity of heat emitted by the unity of surface of a body during the unity of time, be represented by the known function of the temperature, *plus* a constant, so that we have

$$e = \mathrm{B}.f.a^{t+\theta} + \mathrm{C},$$

and let us determine the laws of cooling, by supposing that the inclosure has a complete absorbing power, whilst the globe has only an absorbing power represented by b'.

Let θ be the temperature of the inclosure, e' the quantity of heat which it emits from the unity of surface in the unity of time, and 1 its emissive power: the total quantity of heat lost by the globe will be

$$e\,s;$$

the quantity of heat emitted from the unity of surface of the inclosure being e', the portion which reaches the globe is always

$$e' \sin^2 \omega;$$

the globe only absorbs a portion b', consequently a quantity

$$b'\,e' \sin^2 \omega,$$

and it absorbs together of that which it receives from the entire inclosure,

$$s'\,b'\,e' \sin^2 \omega = s\,b'\,e';$$

its definitive loss is therefore

$$e\,s - s\,b'\,e'.$$

But if we have

$$e = \mathrm{B}f.\,a^{t+\theta} + \mathrm{C}, \qquad e' = \mathrm{B}.\,a^{\theta} + \mathrm{C},$$

this loss of heat becomes

$$s\,\mathrm{B}f\,(a^{t+\theta} - a^{\theta}) + s\,\mathrm{C}\,(1 - f),$$

because the radiating power f of the globe is equal to its absorbing power b'.

Let p be the weight of this globe, c its capacity for heat; for a loss of heat $p\,c$, it loses in temperature 1°; and, for the loss expressed above, it loses in temperature a number of degrees expressed by

$$\frac{s\,\mathrm{B}f}{p\,c}(a^{t+\theta} - a^{\theta}) + \frac{s\,\mathrm{C}}{p\,c}(1 - f).$$

Such is the rapidity of cooling.

For this result to coincide with that of MM. Dulong and Petit, we must have

$$m = \frac{s\,\mathrm{B}f}{p\,c} \text{ and } \mathrm{C} = 0.$$

Now MM. Dulong and Petit having submitted to experiment a silvered thermometer in a blackened inclosure, and the perfect accuracy of the law of cooling having been thus verified in the most exact manner, we see that the constant of emission is necessarily equal to zero in this particular case, and consequently it is always null in all possible cases.

We must therefore, without exception, employ the formula

$$e = \mathrm{B}.f\,a^{t},$$

at least throughout the extent of the scale for which the law of cooling is demonstrated. The constant B is, as I have said, an invariable constant, the numerical value of which depends on the unities of surface and of time which we select and on the point whence we reckon the temperatures, whilst the numerical value of a depends on the number of degrees or divisions which we establish between the two fixed points which serve as a base to the thermometric scale. If, for example, we raised the zero of the scale 100°, preserving for the degree the magnitude which it has actually in the centigrade scale, a would remain equal to 1·0077, and the value of B would have to be multiplied by a^{100}.

With respect to the radiating power f, we may remark that, for athermanous substances, its value only depends absolutely upon the state of the surface, and appears to be comprised between $\frac{1}{10}$ and 1 for all known surfaces: but, in diathermanous substances, f depends both upon the state of the surface and upon various other properties of these substances. It will now be an essential point for the theory of heat to determine what are the properties which may modify the value of f, and the part of the influence which pertains to each of them in these variations.

Note II.

I have made a great number of experiments with the actinometer, exposing it perpendicularly to the solar rays at different hours of the day: its central thermometer then assumes considerable elevations of temperature; it often rises to 50° above the ambient temperature towards noon, and it sometimes even rose to 90°, the temperature of the air being 27°, which gives an elevation of 63°. I point out these results only to show, on the one hand, that the actinometer might be graduated by the pyrheliometer, if we only desired to have approximate results of the solar heat; and to show, on the other hand, that thermometers exposed to the sun may, according to the dispositions given them, take any required excess above the temperature of the air from 3° or 4° up to 63° or 64°.

Article IV.

On the Structure of the Vegetable Cell. By Hugo Mohl.

[From the *Botanische Zeitung*, April 12th, 19th, 26th; May 3rd, 10th*.]

In my phytotomical researches I have frequently met with isolated appearances which were not exactly opposed to the views I had previously advanced, that the septa of cells and vessels consist of two membranes overlying each other,—a primary, external and imperforate, and a secondary generally pierced with apertures,—but which phænomena I did not know how to connect with others previously known to me. An evanescent appearance which I had only once met with, and which at the time made a considerable impression on me, referred particularly to this point. Some years ago, when examining a fresh *Jungermannia* under water, I observed in a cell of one of the leaves, that the chlorophylle granules did not lie upon the wall of the cell, as is usually the case in the *Jungermanniæ*, and also occurs in other cells of leaves, but were united, in the middle of the cell, into a globular mass which suddenly expanded, assumed the form of a thin membranous cell inclosing the chlorophylle granules, and quickly increasing in size, after a few seconds filled the whole space of the leaf-cell, and then could no longer be distinguished from the primary wall, to which it closely applied itself. In this appearance, at first unintelligible to me, a peculiar condition of structure presented itself, a more accurate examination of which baffled my endeavours for a long time. But when Kutzing recently made known that the elementary organs of the Algæ consist of cells inclosed one within another, which view he has since set forth more distinctly and illustrated by beautiful drawings in his 'Phycologia generalis;' when Meneghini, with whom I discussed this view, told me that he also admitted the existence of an inner cell in *Zygnema*, and Hartig† advanced a theory of the mode of development of cellular membrane diametrically opposed to mine, I found myself called upon to submit the cell-wall structure to further observation.

* Translated by Arthur Henfrey, M.R.C.S., F.L.S.

† *Beiträge zur Entwicklungsgeschichte der Pflanzen.* 1843.